Koji Ichidayama, SF J Nuclear Science, 2018, 2:3

SciFed Journal of Nuclear Science

Research Article Open Access

Tensor Satisfying Binary Law about the First, Second, Third-Order Covariant Derivative of the Contravariant Vector

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Abstract

If all coordinate systems satisfies Binary Law, I have already reported that $A^{\mu}_{;\nu} = \frac{\partial A^{\mu}}{\partial x^{\nu}}$ is established for the first-order covariant derivative of the contravariant vector. In addition, if all coordinate systems satisfies Binary Law, I have already reported that $A^{\mu}_{;\nu;\nu} = \frac{\partial^2 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}}$, $A^{\mu}_{;\nu;\nu;\nu} = \frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}$ is established for the second, third-order covariant derivative of the contravariant vector. I report proof put in order based on proof in the above newly in this article. Furthermore, I report "Property of Tensor satisfying Binary Law about the third-order Covariant Derivative of the contravariant Vector" in this article.

Keywords

Tensor; Covariant Derivative of the contravariant Vector; Binary Law; Tensor Satisfying Binary Law

Introduction

I have already reported Definision 9, Definision 10, Definision 11. I remade proof of the derivation of Tensor satisfying Binary Law: $A^{\mu}_{;\nu} = \frac{\partial A^{\mu}}{\partial x^{\nu}}$ based on proof in Definision 9. Similarly, I remade proof of the derivation of Tensor satisfying Binary Law: $A^{\mu}_{;\nu;\nu} = \frac{\partial^2 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}}$ based on proof in Definision10. Similarly, I remade proof of the derivation of Tensor satisfying Binary Law: $A^{\mu}_{;\nu;\nu;\nu} = \frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}$ based on proof in Definision 11. I report the proof provided newly mentioned above in this article.

2 Definision

Definision 1 $\overline{x^{\mu}} \neq x^{\mu}, \overline{x^{\nu}} \neq x^{\nu}, \overline{x^{\mu}} = x^{\nu}, \overline{x^{\nu}} = x^{\mu}$ is established. [1] I named

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Received March 14, 2018; **Accepted** March 29, 2018; **Published** April 12, 2018

Citation: Koji Ichidayama (2018) Tensor Satisfying Binary Law about the First, Second, Third-Order Covariant Derivative of the Contravariant Vector. SF J Nuclear Science 2:3.

$\overline{x^\mu} \neq x^\mu, \overline{x^\nu} \neq x^\nu, \overline{x^\mu} = x^\nu, \overline{x^\nu} = x^\mu$ "Binary Law". [1]

Definision 2 If $\overline{x^\mu} \neq x^\mu, \overline{x^\nu} \neq x^\nu, \overline{x^\mu} = x^\nu, \overline{x^\nu} = x^\mu$ is established, $x^\mu = x_\nu$ is established.[1]

Definision 3 If $\overline{x^\mu} \neq x^\mu, \overline{x^\nu} \neq x^\nu, \overline{x^\mu} = x^\nu, \overline{x^\nu} = x^\mu$ is established, $x_\nu = -x_\mu$ is established.[1]

Definision 4 If all coordinate systems $x^\mu, x^\nu, x^\sigma, x^\lambda, \ldots$ satisfies $\overline{x^\mu} \neq x^\mu, \overline{x^\nu} \neq x^\nu, \overline{x^\mu} = x^\nu, \overline{x^\nu} = x^\mu$, all coordinate systems $x^\mu, x^\nu, x^\sigma, x^\lambda, \ldots$ shifts to only two of x^μ, x^ν. [1]

Definision 5 $g^\mu_\mu = 1, g^\mu_\nu = 0 : (\mu \neq \nu)$ is establishment .[3]

Definision 6 The first-order covariant derivative of the contravariant vector satisfied

$$A^\mu_{;\nu} = \frac{\partial A^\mu}{\partial x^\nu} + A^\tau \Gamma^\mu_{\tau\nu} = \frac{\partial A^\mu}{\partial x^\nu} + A^\tau \frac{1}{2} g^{\in\mu} \left(\frac{\partial g_{\tau\in}}{\partial x^\nu} + \frac{\partial g_{\nu\in}}{\partial x^\tau} - \frac{\partial g_{\tau\nu}}{\partial x^\in} \right) .[4]$$

Definision 7 The second-order covariant derivative of the contravariant vector satisfied

$$\begin{aligned}
A^\mu_{;\nu;\sigma} &= \frac{\partial A^\mu_{;\nu}}{\partial x^\sigma} + A^\iota_{;\nu} \Gamma^\mu_{\iota\sigma} - A^\mu_{;\iota} \Gamma^\iota_{\nu\sigma} \\
&= \frac{\partial}{\partial x^\sigma} \left(\frac{\partial A^\mu}{\partial x^\nu} + A^\tau \Gamma^\mu_{\tau\nu} \right) + \left(\frac{\partial A^\iota}{\partial x^\nu} + A^\tau \Gamma^\iota_{\tau\nu} \right) \Gamma^\mu_{\iota\sigma} - \left(\frac{\partial A^\mu}{\partial x^\iota} + A^\tau \Gamma^\mu_{\tau\iota} \right) \Gamma^\iota_{\nu\sigma} \\
&= \frac{\partial^2 A^\mu}{\partial x^\nu \partial x^\sigma} + \frac{\partial}{\partial x^\sigma} \left(A^\tau \frac{1}{2} g^{\in\mu} \left(\frac{\partial g_{\tau\in}}{\partial x^\nu} + \frac{\partial g_{\nu\in}}{\partial x^\tau} - \frac{\partial g_{\tau\nu}}{\partial x^\in} \right) \right) \\
&+ \frac{\partial A^\iota}{\partial x^\nu} \frac{1}{2} g^{\in\mu} \left(\frac{\partial g_{\iota\in}}{\partial x^\sigma} + \frac{\partial g_{\sigma\in}}{\partial x^\iota} - \frac{\partial g_{\iota\sigma}}{\partial x^\in} \right) + A^\tau \frac{1}{2} g^{\in\iota} \left(\frac{\partial g_{\tau\in}}{\partial x^\nu} + \frac{\partial g_{\nu\in}}{\partial x^\tau} - \frac{\partial g_{\tau\nu}}{\partial x^\in} \right) \frac{1}{2} g^{\in\mu} \left(\frac{\partial g_{\iota\in}}{\partial x^\sigma} + \frac{\partial g_{\sigma\in}}{\partial x^\iota} - \frac{\partial g_{\iota\sigma}}{\partial x^\in} \right) \\
&- \frac{\partial A^\mu}{\partial x^\iota} \frac{1}{2} g^{\in\iota} \left(\frac{\partial g_{\nu\in}}{\partial x^\sigma} + \frac{\partial g_{\sigma\in}}{\partial x^\nu} - \frac{\partial g_{\nu\sigma}}{\partial x^\in} \right) - A^\tau \frac{1}{2} g^{\in\mu} \left(\frac{\partial g_{\tau\in}}{\partial x^\iota} + \frac{\partial g_{\iota\in}}{\partial x^\tau} - \frac{\partial g_{\tau\iota}}{\partial x^\in} \right) \frac{1}{2} g^{\in\iota} \left(\frac{\partial g_{\nu\in}}{\partial x^\sigma} + \frac{\partial g_{\sigma\in}}{\partial x^\nu} - \frac{\partial g_{\nu\sigma}}{\partial x^\in} \right) .[4]
\end{aligned}$$

Definision 8 The third-order covariant derivative of the contravariant vector satisfied

$$A^\mu_{;\nu;\sigma;\lambda} = \frac{\partial A^\mu_{;\nu;\sigma}}{\partial x^\lambda} + A^\kappa_{;\nu;\sigma} \Gamma^\mu_{\kappa\lambda} - A^\mu_{;\kappa;\sigma} \Gamma^\kappa_{\nu\lambda} - A^\mu_{;\nu;\kappa} \Gamma^\kappa_{\sigma\lambda}$$

$$=\frac{\partial}{\partial x^{\lambda}}\left\{\frac{\partial}{\partial x^{\sigma}}\left(\frac{\partial A^{\mu}}{\partial x^{\nu}}+A^{\tau}\Gamma^{\mu}_{\tau\nu}\right)+\left(\frac{\partial A^{\iota}}{\partial x^{\nu}}+A^{\tau}\Gamma^{\iota}_{\tau\nu}\right)\Gamma^{\mu}_{\iota\sigma}-\left(\frac{\partial A^{\mu}}{\partial x^{\iota}}+A^{\tau}\Gamma^{\mu}_{\tau\iota}\right)\Gamma^{\iota}_{\nu\sigma}\right\}$$

$$+\left\{\frac{\partial}{\partial x^{\sigma}}\left(\frac{\partial A^{\kappa}}{\partial x^{\nu}}+A^{\tau}\Gamma^{\kappa}_{\tau\nu}\right)+\left(\frac{\partial A^{\iota}}{\partial x^{\nu}}+A^{\tau}\Gamma^{\iota}_{\tau\nu}\right)\Gamma^{\kappa}_{\iota\sigma}-\left(\frac{\partial A^{\kappa}}{\partial x^{\iota}}+A^{\tau}\Gamma^{\kappa}_{\tau\iota}\right)\Gamma^{\iota}_{\nu\sigma}\right\}\Gamma^{\mu}_{\kappa\lambda}$$

$$-\left\{\frac{\partial}{\partial x^{\sigma}}\left(\frac{\partial A^{\mu}}{\partial x^{\kappa}}+A^{\tau}\Gamma^{\mu}_{\tau\kappa}\right)+\left(\frac{\partial A^{\iota}}{\partial x^{\kappa}}+A^{\tau}\Gamma^{\iota}_{\tau\kappa}\right)\Gamma^{\mu}_{\iota\sigma}-\left(\frac{\partial A^{\mu}}{\partial x^{\iota}}+A^{\tau}\Gamma^{\mu}_{\tau\iota}\right)\Gamma^{\iota}_{\kappa\sigma}\right\}\Gamma^{\kappa}_{\nu\lambda}$$

$$-\left\{\frac{\partial}{\partial x^{\kappa}}\left(\frac{\partial A^{\mu}}{\partial x^{\nu}}+A^{\tau}\Gamma^{\mu}_{\tau\nu}\right)+\left(\frac{\partial A^{\iota}}{\partial x^{\nu}}+A^{\tau}\Gamma^{\iota}_{\tau\nu}\right)\Gamma^{\mu}_{\iota\kappa}-\left(\frac{\partial A^{\mu}}{\partial x^{\iota}}+A^{\tau}\Gamma^{\mu}_{\tau\iota}\right)\Gamma^{\iota}_{\nu\kappa}\right\}\Gamma^{\kappa}_{\sigma\lambda}$$

$$=\frac{\partial^{3}A^{\mu}}{\partial x^{\nu}\partial x^{\sigma}\partial x^{\lambda}}+\frac{\partial^{2}}{\partial x^{\sigma}\partial x^{\lambda}}\left(A^{\tau}\frac{1}{2}g^{\in\mu}\left(\frac{\partial g_{\tau\in}}{\partial x^{\nu}}+\frac{\partial g_{\nu\in}}{\partial x^{\tau}}-\frac{\partial g_{\tau\nu}}{\partial x^{\in}}\right)\right)$$

$$+\frac{\partial}{\partial x^{\lambda}}\left(\frac{\partial A^{\iota}}{\partial x^{\nu}}\frac{1}{2}g^{\in\mu}\left(\frac{\partial g_{\iota\in}}{\partial x^{\sigma}}+\frac{\partial g_{\sigma\in}}{\partial x^{\iota}}-\frac{\partial g_{\iota\sigma}}{\partial x^{\in}}\right)\right)$$

$$+\frac{\partial}{\partial x^{\lambda}}\left(A^{\tau}\frac{1}{2}g^{\in\iota}\left(\frac{\partial g_{\tau\in}}{\partial x^{\nu}}+\frac{\partial g_{\nu\in}}{\partial x^{\tau}}-\frac{\partial g_{\tau\nu}}{\partial x^{\in}}\right)\frac{1}{2}g^{\in\mu}\left(\frac{\partial g_{\iota\in}}{\partial x^{\sigma}}+\frac{\partial g_{\sigma\in}}{\partial x^{\iota}}-\frac{\partial g_{\iota\sigma}}{\partial x^{\in}}\right)\right)$$

$$-\frac{\partial}{\partial x^{\lambda}}\left(\frac{\partial A^{\mu}}{\partial x^{\iota}}\frac{1}{2}g^{\in\iota}\left(\frac{\partial g_{\nu\in}}{\partial x^{\sigma}}+\frac{\partial g_{\sigma\in}}{\partial x^{\nu}}-\frac{\partial g_{\nu\sigma}}{\partial x^{\in}}\right)\right)$$

$$-\frac{\partial}{\partial x^{\lambda}}\left(A^{\tau}\frac{1}{2}g^{\in\mu}\left(\frac{\partial g_{\tau\in}}{\partial x^{\iota}}+\frac{\partial g_{\iota\in}}{\partial x^{\tau}}-\frac{\partial g_{\tau\iota}}{\partial x^{\in}}\right)\frac{1}{2}g^{\in\iota}\left(\frac{\partial g_{\nu\in}}{\partial x^{\sigma}}+\frac{\partial g_{\sigma\in}}{\partial x^{\nu}}-\frac{\partial g_{\nu\sigma}}{\partial x^{\in}}\right)\right)$$

$$+\frac{\partial^{2}A^{\kappa}}{\partial x^{\nu}\partial x^{\sigma}}\frac{1}{2}g^{\in\mu}\left(\frac{\partial g_{\kappa\in}}{\partial x^{\lambda}}+\frac{\partial g_{\lambda\in}}{\partial x^{\kappa}}-\frac{\partial g_{\kappa\lambda}}{\partial x^{\in}}\right)$$

$$+\frac{\partial}{\partial x^{\sigma}}\left(A^{\tau}\frac{1}{2}g^{\in\kappa}\left(\frac{\partial g_{\tau\in}}{\partial x^{\nu}}+\frac{\partial g_{\nu\in}}{\partial x^{\tau}}-\frac{\partial g_{\tau\nu}}{\partial x^{\in}}\right)\right)\frac{1}{2}g^{\in\mu}\left(\frac{\partial g_{\kappa\in}}{\partial x^{\lambda}}+\frac{\partial g_{\lambda\in}}{\partial x^{\kappa}}-\frac{\partial g_{\kappa\lambda}}{\partial x^{\in}}\right)$$

$$+\frac{\partial A^{\iota}}{\partial x^{\nu}}\frac{1}{2}g^{\in\kappa}\left(\frac{\partial g_{\iota\in}}{\partial x^{\sigma}}+\frac{\partial g_{\sigma\in}}{\partial x^{\iota}}-\frac{\partial g_{\iota\sigma}}{\partial x^{\in}}\right)\frac{1}{2}g^{\in\mu}\left(\frac{\partial g_{\kappa\in}}{\partial x^{\lambda}}+\frac{\partial g_{\lambda\in}}{\partial x^{\kappa}}-\frac{\partial g_{\kappa\lambda}}{\partial x^{\in}}\right)$$

$$+A^{\tau}\frac{1}{2}g^{\in\iota}\left(\frac{\partial g_{\tau\in}}{\partial x^{\nu}}+\frac{\partial g_{\nu\in}}{\partial x^{\tau}}-\frac{\partial g_{\tau\nu}}{\partial x^{\in}}\right)\frac{1}{2}g^{\in\kappa}\left(\frac{\partial g_{\iota\in}}{\partial x^{\sigma}}+\frac{\partial g_{\sigma\in}}{\partial x^{\iota}}-\frac{\partial g_{\iota\sigma}}{\partial x^{\in}}\right)\frac{1}{2}g^{\in\mu}\left(\frac{\partial g_{\kappa\in}}{\partial x^{\lambda}}+\frac{\partial g_{\lambda\in}}{\partial x^{\kappa}}-\frac{\partial g_{\kappa\lambda}}{\partial x^{\in}}\right)$$

$$-\frac{\partial A^{\kappa}}{\partial x^{\iota}}\frac{1}{2}g^{\in\iota}\left(\frac{\partial g_{\nu\in}}{\partial x^{\sigma}}+\frac{\partial g_{\sigma\in}}{\partial x^{\nu}}-\frac{\partial g_{\nu\sigma}}{\partial x^{\in}}\right)\frac{1}{2}g^{\in\mu}\left(\frac{\partial g_{\kappa\in}}{\partial x^{\lambda}}+\frac{\partial g_{\lambda\in}}{\partial x^{\kappa}}-\frac{\partial g_{\kappa\lambda}}{\partial x^{\in}}\right)$$

$$-A^{\tau}\frac{1}{2}g^{\in\kappa}\left(\frac{\partial g_{\tau\in}}{\partial x^{\iota}}+\frac{\partial g_{\iota\in}}{\partial x^{\tau}}-\frac{\partial g_{\tau\iota}}{\partial x^{\in}}\right)\frac{1}{2}g^{\in\iota}\left(\frac{\partial g_{\nu\in}}{\partial x^{\sigma}}+\frac{\partial g_{\sigma\in}}{\partial x^{\nu}}-\frac{\partial g_{\nu\sigma}}{\partial x^{\in}}\right)\frac{1}{2}g^{\in\mu}\left(\frac{\partial g_{\kappa\in}}{\partial x^{\lambda}}+\frac{\partial g_{\lambda\in}}{\partial x^{\kappa}}-\frac{\partial g_{\kappa\lambda}}{\partial x^{\in}}\right)$$

$$-\frac{\partial^{2}A^{\mu}}{\partial x^{\kappa}\partial x^{\sigma}}\frac{1}{2}g^{\in\kappa}\left(\frac{\partial g_{\nu\in}}{\partial x^{\lambda}}+\frac{\partial g_{\lambda\in}}{\partial x^{\nu}}-\frac{\partial g_{\nu\lambda}}{\partial x^{\in}}\right)$$

$$-\frac{\partial}{\partial x^{\sigma}}\left(A^{\tau}\frac{1}{2}g^{\in\mu}\left(\frac{\partial g_{\tau\in}}{\partial x^{\kappa}}+\frac{\partial g_{\kappa\in}}{\partial x^{\tau}}-\frac{\partial g_{\tau\kappa}}{\partial x^{\in}}\right)\right)\frac{1}{2}g^{\in\kappa}\left(\frac{\partial g_{\nu\in}}{\partial x^{\lambda}}+\frac{\partial g_{\lambda\in}}{\partial x^{\nu}}-\frac{\partial g_{\nu\lambda}}{\partial x^{\in}}\right)$$

$$-\frac{\partial A^{\iota}}{\partial x^{\kappa}}\frac{1}{2}g^{\in\mu}\left(\frac{\partial g_{\iota\in}}{\partial x^{\sigma}}+\frac{\partial g_{\sigma\in}}{\partial x^{\iota}}-\frac{\partial g_{\iota\sigma}}{\partial x^{\in}}\right)\frac{1}{2}g^{\in\kappa}\left(\frac{\partial g_{\nu\in}}{\partial x^{\lambda}}+\frac{\partial g_{\lambda\in}}{\partial x^{\nu}}-\frac{\partial g_{\nu\lambda}}{\partial x^{\in}}\right)$$

$$-A^{\tau}\frac{1}{2}g^{\in\iota}\left(\frac{\partial g_{\tau\in}}{\partial x^{\kappa}}+\frac{\partial g_{\kappa\in}}{\partial x^{\tau}}-\frac{\partial g_{\tau\kappa}}{\partial x^{\in}}\right)\frac{1}{2}g^{\in\mu}\left(\frac{\partial g_{\iota\in}}{\partial x^{\sigma}}+\frac{\partial g_{\sigma\in}}{\partial x^{\iota}}-\frac{\partial g_{\iota\sigma}}{\partial x^{\in}}\right)\frac{1}{2}g^{\in\kappa}\left(\frac{\partial g_{\nu\in}}{\partial x^{\lambda}}+\frac{\partial g_{\lambda\in}}{\partial x^{\nu}}-\frac{\partial g_{\nu\lambda}}{\partial x^{\in}}\right)$$
$$+\frac{\partial A^{\mu}}{\partial x^{\iota}}\frac{1}{2}g^{\in\iota}\left(\frac{\partial g_{\kappa\in}}{\partial x^{\sigma}}+\frac{\partial g_{\sigma\in}}{\partial x^{\kappa}}-\frac{\partial g_{\kappa\sigma}}{\partial x^{\in}}\right)\frac{1}{2}g^{\in\kappa}\left(\frac{\partial g_{\nu\in}}{\partial x^{\lambda}}+\frac{\partial g_{\lambda\in}}{\partial x^{\nu}}-\frac{\partial g_{\nu\lambda}}{\partial x^{\in}}\right)$$
$$+A^{\tau}\frac{1}{2}g^{\in\mu}\left(\frac{\partial g_{\tau\in}}{\partial x^{\iota}}+\frac{\partial g_{\iota\in}}{\partial x^{\tau}}-\frac{\partial g_{\tau\iota}}{\partial x^{\in}}\right)\frac{1}{2}g^{\in\iota}\left(\frac{\partial g_{\kappa\in}}{\partial x^{\sigma}}+\frac{\partial g_{\sigma\in}}{\partial x^{\kappa}}-\frac{\partial g_{\kappa\sigma}}{\partial x^{\in}}\right)\frac{1}{2}g^{\in\kappa}\left(\frac{\partial g_{\nu\in}}{\partial x^{\lambda}}+\frac{\partial g_{\lambda\in}}{\partial x^{\nu}}-\frac{\partial g_{\nu\lambda}}{\partial x^{\in}}\right)$$
$$-\frac{\partial^2 A^{\mu}}{\partial x^{\nu}\partial x^{\kappa}}\frac{1}{2}g^{\in\kappa}\left(\frac{\partial g_{\sigma\in}}{\partial x^{\lambda}}+\frac{\partial g_{\lambda\in}}{\partial x^{\sigma}}-\frac{\partial g_{\sigma\lambda}}{\partial x^{\in}}\right)$$
$$-\frac{\partial}{\partial x^{\kappa}}\left(A^{\tau}\frac{1}{2}g^{\in\mu}\left(\frac{\partial g_{\tau\in}}{\partial x^{\nu}}+\frac{\partial g_{\nu\in}}{\partial x^{\tau}}-\frac{\partial g_{\tau\nu}}{\partial x^{\in}}\right)\right)\frac{1}{2}g^{\in\kappa}\left(\frac{\partial g_{\sigma\in}}{\partial x^{\lambda}}+\frac{\partial g_{\lambda\in}}{\partial x^{\sigma}}-\frac{\partial g_{\sigma\lambda}}{\partial x^{\in}}\right)$$
$$-\frac{\partial A^{\iota}}{\partial x^{\nu}}\frac{1}{2}g^{\in\mu}\left(\frac{\partial g_{\iota\in}}{\partial x^{\kappa}}+\frac{\partial g_{\kappa\in}}{\partial x^{\iota}}-\frac{\partial g_{\iota\kappa}}{\partial x^{\in}}\right)\frac{1}{2}g^{\in\kappa}\left(\frac{\partial g_{\sigma\in}}{\partial x^{\lambda}}+\frac{\partial g_{\lambda\in}}{\partial x^{\sigma}}-\frac{\partial g_{\sigma\lambda}}{\partial x^{\in}}\right)$$
$$-A^{\tau}\frac{1}{2}g^{\in\iota}\left(\frac{\partial g_{\tau\in}}{\partial x^{\nu}}+\frac{\partial g_{\nu\in}}{\partial x^{\tau}}-\frac{\partial g_{\tau\nu}}{\partial x^{\in}}\right)\frac{1}{2}g^{\in\mu}\left(\frac{\partial g_{\iota\in}}{\partial x^{\kappa}}+\frac{\partial g_{\kappa\in}}{\partial x^{\iota}}-\frac{\partial g_{\iota\kappa}}{\partial x^{\in}}\right)\frac{1}{2}g^{\in\kappa}\left(\frac{\partial g_{\sigma\in}}{\partial x^{\lambda}}+\frac{\partial g_{\lambda\in}}{\partial x^{\sigma}}-\frac{\partial g_{\sigma\lambda}}{\partial x^{\in}}\right)$$
$$+\frac{\partial A^{\mu}}{\partial x^{\iota}}\frac{1}{2}g^{\in\iota}\left(\frac{\partial g_{\nu\in}}{\partial x^{\kappa}}+\frac{\partial g_{\kappa\in}}{\partial x^{\nu}}-\frac{\partial g_{\nu\kappa}}{\partial x^{\in}}\right)\frac{1}{2}g^{\in\kappa}\left(\frac{\partial g_{\sigma\in}}{\partial x^{\lambda}}+\frac{\partial g_{\lambda\in}}{\partial x^{\sigma}}-\frac{\partial g_{\sigma\lambda}}{\partial x^{\in}}\right)$$
$$+A^{\tau}\frac{1}{2}g^{\in\mu}\left(\frac{\partial g_{\tau\in}}{\partial x^{\iota}}+\frac{\partial g_{\iota\in}}{\partial x^{\tau}}-\frac{\partial g_{\tau\iota}}{\partial x^{\in}}\right)\frac{1}{2}g^{\in\iota}\left(\frac{\partial g_{\nu\in}}{\partial x^{\kappa}}+\frac{\partial g_{\kappa\in}}{\partial x^{\nu}}-\frac{\partial g_{\nu\kappa}}{\partial x^{\in}}\right)\frac{1}{2}g^{\in\kappa}\left(\frac{\partial g_{\sigma\in}}{\partial x^{\lambda}}+\frac{\partial g_{\lambda\in}}{\partial x^{\sigma}}-\frac{\partial g_{\sigma\lambda}}{\partial x^{\in}}\right)$$.[4]

Definision 9 If all coordinate systems $x^{\mu},x^{\nu},x^{\sigma},x^{\lambda},...$ satisfies $\overline{x^{\mu}}\neq x^{\mu},\overline{x^{\nu}}\neq x^{\nu},\overline{x^{\mu}}=x^{\nu},\overline{x^{\nu}}=x^{\mu}$**,** convariant differentiation for Contravariant Vector $A^{\mu}_{;\nu}$ behave like a convariant differentiation for Scalar $S_{;\nu}$. [1]

Definision 10 If $\overline{x^{\mu}}\neq x^{\mu}\ \overline{x^{\nu}}\neq x^{\nu},\overline{x^{\mu}}=x^{\nu},\overline{x^{\nu}}=x^{\mu}$ is established, $A^{\mu}_{;\nu;\nu}=\dfrac{\partial^2 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}}$ is established .[2]

Definision 11 If $\overline{x^{\mu}}\neq x^{\mu}\ \overline{x^{\nu}}\neq x^{\nu},\overline{x^{\mu}}=x^{\nu},\overline{x^{\nu}}=x^{\mu}$ is established, $A^{\mu}_{;\nu;\nu;\nu}=\dfrac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}$ is established .[2]

Definision 12 I express x^{μ} satisfying $\dfrac{\partial^3 x^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}=M\ in\left\{x^{\mu}\right\}_M$.

3 Tensor satisfying Binary Law about the first-order Covariant Derivative of the Contravariant Vector

Proposition 1 Definision6 isn't an equation of the tensor satisfying Binary Law.

Proof: If all coordinate systems satisfies Binary Law, I get

$$\begin{aligned}A^{\mu}_{;\nu}&=\frac{\partial A^{\mu}}{\partial x^{\nu}}+A^{\nu}\frac{1}{2}g^{\nu\mu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\\&=\frac{\partial A^{\mu}}{\partial x^{\nu}}+A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\right)\end{aligned}\quad(1)$$

From Definision 6. I rewrite dummy index of (1) in the index which isn't used in this term and must get

$$A^{\mu}_{;\nu}=\frac{\partial A^{\mu}}{\partial x^{\nu}}+A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\nu}}\right)\quad,(2)$$

$$A^{\mu}_{;\nu} = \frac{\partial A^{\mu}}{\partial x^{\nu}} + A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\sigma}}\right) \qquad (3)$$

because (1) must be an equation of the tensor. I get the conclusion that (2), (3) doesn't satisfy Binary law from Definision4. If (2), (3) satisfies Binary Law, Definision 6 is an equation of the tensor satisfying Binary Law here.However, I get the conclusion that Definision6 isn't an equation of the tensor satisfyingBinary Law because (2), (3) doesn't satisfy Binary Law.

– End Proof –

Proposition 2 $A^{\mu}_{;\nu} = \frac{\partial A^{\mu}}{\partial x^{\nu}}$ is the equation that was rewritten so that Definision 6 is an equation of the tensor satisfying Binary Law.

Proof: I rewrite index ν of the coordinate systems of (2), (3) by Definision 3 and get

$$-A^{\mu}_{;\mu} = -\frac{\partial A^{\mu}}{\partial x^{\mu}} - A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right), \qquad (4)$$

$$-A^{\mu}_{;\mu} = -\frac{\partial A^{\mu}}{\partial x^{\mu}} - A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right). \qquad (5)$$

There isn't index ν in the second term of the right side of (4), (5) at all here. Therefore, I rewrite dummy index σ of (4), (5) in dummy index ν and can get

$$-A^{\mu}_{;\mu} = -\frac{\partial A^{\mu}}{\partial x^{\mu}} - A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\right), \qquad (6)$$

$$-A^{\mu}_{;\mu} = -\frac{\partial A^{\mu}}{\partial x^{\mu}} - A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}\right). \qquad (7)$$

And I get

$$-A^{\mu}_{;\mu} = -\frac{\partial A^{\mu}}{\partial x^{\mu}} \qquad (8)$$

from (6), (7) in consideration of Definision 5. In addition,

$$A^{\mu}_{;\nu} = \frac{\partial A^{\mu}}{\partial x^{\nu}} \qquad (9)$$

can rewrite (8) using Definision 2 because the second term doesn't exist in the right side of (8). Furthermore, I rewrite index ν of the coordinate systems of (2), (3) by Definision 2 and get

$$A^{\mu;\mu} = \frac{\partial A^{\mu}}{\partial x_{\mu}} + A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right), \qquad (10)$$

$$A^{\mu;\mu} = \frac{\partial A^{\mu}}{\partial x_{\mu}} + A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right). \qquad (11)$$

Because two same cotravariant index existed in $A^{\mu;\mu}$ of (10), (11), I decided not to handle (10), (11). Because (9) doesn't change the form of the equation even if all coordinate systems satisfies Binary Law, I get the conclusion that (9) satisfies Binary Law. In other words, (9) is an equation of the tensor satisfying Binary Law. Because (9) is an equation provided from Definision 6, (9) is the equation that was rewritten so that Definision 6 is an equation of the tensor satisfying Binary Law.

– End Proof –

4 Tensor satisfying Binary Law about the second-order Covariant Derivative of the Contravariant Vector

Proposition 3 Definision 7 isn't an equation of the tensor satisfying Binary Law.

Proof: If all coordinate systems satisfies Binary Law, I get

$$
\begin{aligned}
A^{\mu}_{;\nu;\nu} = & \frac{\partial^2 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}} \\
& + \frac{\partial}{\partial x^{\nu}}\left(A^{\nu}\frac{1}{2}g^{\nu\mu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\right) \\
& + \frac{\partial A^{\nu}}{\partial x^{\nu}}\frac{1}{2}g^{\nu\mu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right) \\
& + A^{\nu}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\mu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right) \\
& - \frac{\partial A^{\mu}}{\partial x^{\nu}}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right) \\
& - A^{\nu}\frac{1}{2}g^{\nu\mu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right) \quad (12)
\end{aligned}
$$

$$
\begin{aligned}
= & \frac{\partial^2 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}} \\
& + \frac{\partial}{\partial x^{\nu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\right)\right) \quad (13) \\
& + \frac{\partial A^{\nu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\right) \quad (14) \\
& - \frac{\partial A^{\mu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\nu}}\right) \quad (15)
\end{aligned}
$$

from Definision 7. I rewrite dummy index of (13) in the index which isn't used in this term and must get

$$A^{\mu}_{;\nu;\nu} = \frac{\partial^2 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}} + \frac{\partial}{\partial x^{\nu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\nu}}\right)\right)\ldots, \quad (16)$$

$$A^{\mu}_{;\nu;\nu} = \frac{\partial^2 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}} + \frac{\partial}{\partial x^{\nu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\sigma}}\right)\right)\ldots, \quad (17)$$

$$A^{\mu}_{;\nu;\nu} = \frac{\partial^2 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}} + \frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\right)\right)\ldots \quad (18)$$

because (12) must be an equation of the tensor. I get the conclusion that (16), (17), (18) doesn't satisfy Binary Law from

Definision 4. Similarly, I rewrite dummy index of (14) in the index which isn't used in this term and must get

$$A^{\mu}_{;\nu;\nu}=\frac{\partial^2 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}}+\frac{\partial A^{\sigma}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\nu}}\right)..., \quad (19)$$

$$A^{\mu}_{;\nu;\nu}=\frac{\partial^2 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}}+\frac{\partial A^{\sigma}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\sigma}}\right)..., \quad (20)$$

$$A^{\mu}_{;\nu;\nu}=\frac{\partial^2 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}}+\frac{\partial A^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\right)... \quad (21)$$

because (12) must be an equation of the tensor. I get the conclusion that (19), (20), (21) doesn't satisfy Binary Law from Definision 4. Similarly, I rewrite dummy index of (15) in the index which isn't used in this term and must get

$$A^{\mu}_{;\nu;\nu}=\frac{\partial^2 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}}-\frac{\partial A^{\mu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\nu}}\right)..., \quad (22)$$

$$A^{\mu}_{;\nu;\nu}=\frac{\partial^2 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}}-\frac{\partial A^{\mu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\sigma}}\right)..., \quad (23)$$

$$A^{\mu}_{;\nu;\nu}=\frac{\partial^2 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}}-\frac{\partial A^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\nu}}\right)... \quad (24)$$

because (12) must be an equation of the tensor. I get the conclusion that (22), (23), (24) doesn't satisfy Binary Law from Definision4. If (16) ~ (24) satisfies Binary Law, Definision 7 is an equation of the tensor satisfying Binary Law here. However, I get the conclusion that Definision 7 isn't an equation of the tensor satisfying Binary Law because (16) ~ (24) doesn't satisfy Binary Law.

–End Proof–

Proposition 4 $A^{\mu}_{;\nu;\nu}=\frac{\partial^2 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}}$ is the equation that was rewritten so that Definision 7 is an equation of the tensor satisfying Binary Law.

Proof: I rewrite index ν of the coordinate systems of (16), (17), (18) by Definision 3 and get

$$A^{\mu}_{;\mu;\mu}=\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)\right)..., \quad (25)$$

$$A^{\mu}_{;\mu;\mu}=\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)\right)..., \quad (26)$$

$$A^{\mu}_{;\mu;\mu}=\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)\right).... \quad (27)$$

There isn't index ν in the second term of the right side of (25), (26), (27) at all here. Therefore, I rewrite dummy index σ of (25), (26), (27) in dummy index ν and can get

$$A^{\mu}_{;\mu;\mu}=\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\right)\right)..., \quad (28)$$

$$A^{\mu}_{;\mu;\mu}=\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}\right)\right)..., \quad (29)$$

$$A^{\mu}_{;\mu;\mu} = \frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}} + \frac{\partial}{\partial x^{\nu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)\right).... \quad (30)$$

Furthermore, I rewrite index ν of the coordinate systems of (16), (17), (18) by Definision 2 and get

$$A^{\mu;\mu;\mu} = \frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}} + \frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)\right)..., \quad (31)$$

$$A^{\mu;\mu;\mu} = \frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}} + \frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)\right)..., \quad (32)$$

$$A^{\mu;\mu;\mu} = \frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}} + \frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x_{\mu}}\right)\right).... \quad (33)$$

Because two same contravariant index existed in $A^{\mu;\mu;\mu}$ of (31), (32), (33), I decided not to handle (31), (32), (33). Furthermore, I rewrite index ν of the coordinate systems of (16), (17), (18) by Definision 3, Definision 2 and get

$$-\left(A^{\mu}_{;\mu}\right)^{;\mu} = -\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}} - \frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)\right)..., \quad (34)$$

$$-\left(A^{\mu}_{;\mu}\right)^{;\mu} = -\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}} - \frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)\right)..., \quad (35)$$

$$-\left(A^{\mu}_{;\mu}\right)^{;\mu} = -\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}} - \frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)\right).... \quad (36)$$

There isn't index ν in the second term of the right side of (34), (35), (36) at all here. Therefore, I rewrite dummy index σ of (34), (35), (36) in dummy index ν and can get

$$-\left(A^{\mu}_{;\mu}\right)^{;\mu} = -\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}} - \frac{\partial}{\partial x_{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\right)\right)..., \quad (37)$$

$$-\left(A^{\mu}_{;\mu}\right)^{;\mu} = -\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}} - \frac{\partial}{\partial x_{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}\right)\right)..., \quad (38)$$

$$-\left(A^{\mu}_{;\mu}\right)^{;\mu} = -\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}} - \frac{\partial}{\partial x^{\nu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)\right).... \quad (39)$$

Furthermore, I rewrite index ν of the coordinate systems of (16), (17), (18) by Definision 2, Definision 3 and get

$$-\left(A^{\mu;\mu}\right)_{;\mu} = -\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}} - \frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)\right)..., \quad (40)$$

$$-\left(A^{\mu;\mu}\right)_{;\mu} = -\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}} - \frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)\right)..., \quad (41)$$

$$-\left(A^{\mu;\mu}\right)_{;\mu} = -\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}} - \frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\mu}}\right)\right).... \quad (42)$$

There isn't index ν in the second term of the right side of (40), (41), (42) at all here. Therefore, I rewrite dummy index σ of (40), (41), (42) in dummy index ν and can get

$$-\left(A^{\mu;\mu}\right)_{;\mu}=-\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}}-\frac{\partial}{\partial x^{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g_{\nu}^{\mu}}{\partial x_{\mu}}\right)\right)..., \quad (43)$$

$$-\left(A^{\mu;\mu}\right)_{;\mu}=-\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}}-\frac{\partial}{\partial x^{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\nu}}\right)\right)..., \quad (44)$$

$$-\left(A^{\mu;\mu}\right)_{;\mu}=-\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}}-\frac{\partial}{\partial x^{\nu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\mu}}\right)\right).... \quad (45)$$

By the way, by establishment of $A^{\mu}_{;\dot{\nu};\ddot{\nu}}=A^{\mu}_{;\ddot{\nu};\dot{\nu}}$, (37) and (43) must be equal. (37) and (43) are equal in consideration of Definision 5. Furthermore, by establishment of $A^{\mu}_{;\dot{\nu};\ddot{\nu}}=A^{\mu}_{;\ddot{\nu};\dot{\nu}}$, (38) and (44) must be equal. (38) and (44) aren't equal in consideration of Definision 5. Thus, I decide not to handle (38), (44). Furthermore, by establishment of $A^{\mu}_{;\dot{\nu};\ddot{\nu}}=A^{\mu}_{;\ddot{\nu};\dot{\nu}}$,(39) and (45) must be equal. (39) and (45) aren't equal in consideration of Definision 5. Thus, I decide not to handle (39), (45).

Similarly . I rewrite index ν of the coordinate systems of (19), (20), (21) by Definision 3 and get

$$A^{\mu}_{;\mu;\mu}=\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}}+\frac{\partial A^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g_{\sigma}^{\mu}}{\partial x^{\mu}}\right)..., \quad (46)$$

$$A^{\mu}_{;\mu;\mu}=\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}}+\frac{\partial A^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g_{\mu}^{\mu}}{\partial x^{\sigma}}\right)..., \quad (47)$$

$$A^{\mu}_{;\mu;\mu}=\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}}+\frac{\partial A^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g_{\mu}^{\mu}}{\partial x^{\mu}}\right).... \quad (48)$$

There isn't index ν in the second term of the right side of (46), (47), (48) at all here. Therefore, I rewrite dummy index σ of (46), (47), (48) in dummy index ν and can get

$$A^{\mu}_{;\mu;\mu}=\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}}+\frac{\partial A^{\nu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g_{\nu}^{\mu}}{\partial x^{\mu}}\right)..., \quad (49)$$

$$A^{\mu}_{;\mu;\mu}=\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}}+\frac{\partial A^{\nu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g_{\mu}^{\mu}}{\partial x^{\nu}}\right)..., \quad (50)$$

$$A^{\mu}_{;\mu;\mu}=\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}}+\frac{\partial A^{\nu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g_{\mu}^{\mu}}{\partial x^{\mu}}\right).... \quad (51)$$

Furthermore, I rewrite index ν of the coordinate systems of (19), (20), (21) by Definision 2 and get

$$A^{\mu;\mu;\mu}=\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}}+\frac{\partial A^{\sigma}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g_{\sigma}^{\mu}}{\partial x_{\mu}}\right)..., \quad (52)$$

$$A^{\mu;\mu;\mu}=\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}}+\frac{\partial A^{\sigma}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)..., \quad (53)$$

$$A^{\mu;\mu;\mu}=\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}}+\frac{\partial A^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x_{\mu}}\right).... \quad (54)$$

Because two same contravariant index existed in $A^{\mu;\mu;\mu}$ of (52), (53), (54), I decided not to handle (52), (53), (54). Furthermore, I rewrite index ν of the coordinate systems of (19), (20), (21) by Definision 3, Definision 2 and get

$$-\left(A^{\mu}_{;\mu}\right)^{;\mu}=-\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}}-\frac{\partial A^{\sigma}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)\ldots,\quad(55)$$

$$-\left(A^{\mu}_{;\mu}\right)^{;\mu}=-\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}}-\frac{\partial A^{\sigma}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)\ldots,\quad(56)$$

$$-\left(A^{\mu}_{;\mu}\right)^{;\mu}=-\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}}-\frac{\partial A^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)\ldots.\quad(57)$$

There isn't index ν in the second term of the right side of (55), (56), (57) at all here. Therefore, I rewrite dummy index σ of (55), (56), (57) in dummy index ν and can get

$$-\left(A^{\mu}_{;\mu}\right)^{;\mu}=-\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}}-\frac{\partial A^{\nu}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\right)\ldots,\quad(58)$$

$$-\left(A^{\mu}_{;\mu}\right)^{;\mu}=-\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}}-\frac{\partial A^{\nu}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}\right)\ldots,\quad(59)$$

$$-\left(A^{\mu}_{;\mu}\right)^{;\mu}=-\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}}-\frac{\partial A^{\nu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)\ldots.\quad(60)$$

Furthermore, I rewrite index ν of the coordinate systems of (19), (20), (21) by Definision 2, Definision 3 and get

$$-\left(A^{\mu;\mu}\right)_{;\mu}=-\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}}-\frac{\partial A^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)\ldots,\quad(61)$$

$$-\left(A^{\mu;\mu}\right)_{;\mu}=-\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}}-\frac{\partial A^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)\ldots,\quad(62)$$

$$-\left(A^{\mu;\mu}\right)_{;\mu}=-\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}}-\frac{\partial A^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\mu}}\right)\ldots.\quad(63)$$

There isn't index ν in the second term of the right side of (61), (62), (63) at all here. Therefore, I rewrite dummy index σ of (61), (62), (63) in dummy index ν and can get

$$-\left(A^{\mu;\mu}\right)_{;\mu}=-\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}}-\frac{\partial A^{\nu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x_{\mu}}\right)\ldots,\quad(64)$$

$$-\left(A^{\mu;\mu}\right)_{;\mu}=-\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}}-\frac{\partial A^{\nu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\nu}}\right)\ldots,\quad(65)$$

$$-\left(A^{\mu;\mu}\right)_{;\mu}=-\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}}-\frac{\partial A^{\nu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\mu}}\right)\ldots.\quad(66)$$

By the way, by establishment of $A^{\ \mu}_{;\dot{\nu};\ddot{\nu}}=A^{\mu}_{;\ddot{\nu};\dot{\nu}}$, (58) and (64) must be equal. (58) and (64) are equal in consideration of Definision 5. Furthermore, by establishment of $A^{\ \mu}_{;\dot{\nu};\ddot{\nu}}=A^{\mu}_{;\ddot{\nu};\dot{\nu}}$,(59) and (65) must be equal. (59) and (65) aren't equal in consideration of Definision 5. Thus, I decide not to handle (59), (65). Furthermore, by establishment of $A^{\ \mu}_{;\dot{\nu};\ddot{\nu}}=A^{\mu}_{;\ddot{\nu};\dot{\nu}}$,(60) and (66) must be equal. (60) and (66) aren't equal in consideration of Definision 5. Thus, I decide not to handle (60), (66).

Similarly, I rewrite index ν of the coordinate systems of (22), (23), (24) by Definision 3 and get

$$A^{\mu}_{;\mu;\mu}=\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}}-\frac{\partial A^{\mu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)...,\quad (67)$$

$$A^{\mu}_{;\mu;\mu}=\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}}-\frac{\partial A^{\mu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)...,\quad (68)$$

$$A^{\mu}_{;\mu;\mu}=\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}}-\frac{\partial A^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}\right)....\quad (69)$$

There isn't index ν in the second term of the right side of (67), (68), (69) at all here. Therefore, I rewrite dummy index σ of (67), (68), (69) in dummy index ν and can get

$$A^{\mu}_{;\mu;\mu}=\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}}-\frac{\partial A^{\mu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\right)...,\quad (70)$$

$$A^{\mu}_{;\mu;\mu}=\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}}-\frac{\partial A^{\mu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\right)...,\quad (71)$$

$$A^{\mu}_{;\mu;\mu}=\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}}-\frac{\partial A^{\mu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu}}\right)....\quad (72)$$

Furthermore, I rewrite index ν of the coordinate systems of (22), (23), (24) by Definision 2 and get

$$A^{\mu;\mu;\mu}=\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}}-\frac{\partial A^{\mu}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_{\mu}}\right)...,\quad (73)$$

$$A^{\mu;\mu;\mu}=\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}}-\frac{\partial A^{\mu}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\sigma}}\right)...,\quad (74)$$

$$A^{\mu;\mu;\mu}=\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}}-\frac{\partial A^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_{\mu}}\right)....\quad (75)$$

Because two same contravariant index existed in $A^{\mu;\mu;\mu}$ of (73),(74),(75), I decided not to handle (73),(74),(75). Furthermore, I rewrite index ν of the coordinate systems of (22),(23),(24) by Definision3, Definision2 and get

$$-(A^{\mu}_{;\mu})^{;\mu}=-\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}}+\frac{\partial A^{\mu}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)...,\quad (76)$$

$$-(A^{\mu}_{;\mu})^{;\mu}=-\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}}+\frac{\partial A^{\mu}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)...,\quad (77)$$

$$-(A^{\mu}_{;\mu})^{;\mu}=-\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}}+\frac{\partial A^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x_{\mu}}\right)....\quad (78)$$

There isn't index ν in the second term of the right side of (76),(77),(78) at all here. Therefore, I rewrite dummy index σ of (76),(77),(78) in dummy index ν and can get

$$-(A^{\mu}_{;\mu})^{;\mu}=-\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}}+\frac{\partial A^{\mu}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\right)...,\quad (79)$$

$$-(A^{\mu}_{;\mu})^{;\mu}=-\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}}+\frac{\partial A^{\mu}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\right)...,\quad (80)$$

$$-(A^{\mu}_{;\mu})^{;\mu} = -\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}} + \frac{\partial A^{\mu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x_{\mu}}\right).... \quad (81)$$

Furthermore, I rewrite index ν of the coordinate systems of (22), (23), (24) by Definision 2, Definision 3 and get

$$-(A^{\mu;\mu})_{;\mu} = -\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}} + \frac{\partial A^{\mu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_{\mu}}\right)..., \quad (82)$$

$$-(A^{\mu;\mu})_{;\mu} = -\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}} + \frac{\partial A^{\mu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\sigma}}\right)..., \quad (83)$$

$$-(A^{\mu;\mu})_{;\mu} = -\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}} + \frac{\partial A^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\mu}}\right).... \quad (84)$$

There isn't index ν in the second term of the right side of (82), (83), (84) at all here. Therefore, I rewrite dummy index σ of (82), (83), (84) in dummy index ν and can get

$$-(A^{\mu;\mu})_{;\mu} = -\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}} + \frac{\partial A^{\mu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x_{\mu}}\right)..., \quad (85)$$

$$-(A^{\mu;\mu})_{;\mu} = -\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}} + \frac{\partial A^{\mu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^{\nu}}\right)..., \quad (86)$$

$$-(A^{\mu;\mu})_{;\mu} = -\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}} + \frac{\partial A^{\mu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^{\mu}}\right).... \quad (87)$$

By the way, by establishment of $A^{\mu}_{;\dot{\nu};\ddot{\nu}} = A^{\mu}_{;\ddot{\nu};\dot{\nu}}$, (79) and (85) must be equal. (79) and (85) are equal in consideration of Definision 5. Furthermore, by establishment of $A^{\mu}_{;\dot{\nu};\ddot{\nu}} = A^{\mu}_{;\ddot{\nu};\dot{\nu}}$, (80) and (86) must be equal. (80) and (86) aren't equal in consideration of Definision5. Thus, I decide not to handle (80), (86). Furthermore, by establishment of $A^{\mu}_{;\dot{\nu};\ddot{\nu}} = A^{\mu}_{;\ddot{\nu};\dot{\nu}}$, (81) and (87) must be equal. (81) and (87) aren't equal in consideration of Definision 5. Thus, I decide not to handle (81), (87). And I get

$$A^{\mu}_{;\mu;\mu} = \frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}} \quad (88)$$

from (28), (29), (30), (49), (50), (51), (70), (71), (72) in consideration of Definision 5. And I get

$$-(A^{\mu}_{;\mu})^{;\mu} = -\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}} \quad (89)$$

from (37), (58), (79) in consideration of Definision 5. And I get

$$-(A^{\mu;\mu})_{;\mu} = -\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}} \quad (90)$$

from (43), (64), (85) in consideration of Definision 5. In addition,

$$A^{\mu}_{;\nu;\nu} = \frac{\partial^2 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}} \quad (91)$$

can rewrite (88), (89), (90) using Definision 2, Definision 3 because the second term doesn't exist in the right side of (88), (89), (90). Because (91) doesn't change the form of the equation even if all coordinate systems satisfies Binary Law, I get the conclusion that (91) satisfies Binary Law. In other words, (91) is an equation of the tensor satisfying Binary Law. Because (91) is an equation provided from Definision 7, (91) is the equation that was rewritten so that

Definision7 is an equation of the tensor satisfying Binary Law.

– End Proof –

5 Tensor satisfying Binary Law about the third-order Covariant Derivative of the Contravariant Vector

Proposition 5 Definision 8 isn't an equation of the tensor satisfying Binary Law.

Proof: If all coordinate systems satisfies Binary Law, I get

$$
\begin{aligned}
A^{\mu}_{;\nu;\nu;\nu} &= \frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}} \\
&+\frac{\partial^2}{\partial x^{\nu}\partial x^{\nu}}\left(A^{\nu}\frac{1}{2}g^{\nu\mu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\right) \\
&+\frac{\partial}{\partial x^{\nu}}\left(\frac{\partial A^{\nu}}{\partial x^{\nu}}\frac{1}{2}g^{\nu\mu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\right) \\
&+\frac{\partial}{\partial x^{\nu}}\left(A^{\nu}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\mu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\right) \\
&-\frac{\partial}{\partial x^{\nu}}\left(\frac{\partial A^{\mu}}{\partial x^{\nu}}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\right) \\
&-\frac{\partial}{\partial x^{\nu}}\left(A^{\nu}\frac{1}{2}g^{\nu\mu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\right) \\
&+\frac{\partial^2 A^{\nu}}{\partial x^{\nu}\partial x^{\nu}}\frac{1}{2}g^{\nu\mu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right) \\
&+\frac{\partial}{\partial x^{\nu}}\left(A^{\nu}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\right)\frac{1}{2}g^{\nu\mu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right) \\
&+\frac{\partial A^{\nu}}{\partial x^{\nu}}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\mu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right) \\
&+A^{\nu}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\mu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right) \\
&-A^{\nu}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\mu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right) \\
&-\frac{\partial^2 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right) \\
&-\frac{\partial}{\partial x^{\nu}}\left(A^{\nu}\frac{1}{2}g^{\nu\mu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right) \\
&-\frac{\partial A^{\nu}}{\partial x^{\nu}}\frac{1}{2}g^{\nu\mu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right) \\
&-\frac{\partial A^{\nu}}{\partial x^{\nu}}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\mu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)
\end{aligned}
$$

$$-A^{\nu}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\mu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)$$

$$+\frac{\partial A^{\mu}}{\partial x^{\nu}}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)$$

$$+A^{\nu}\frac{1}{2}g^{\nu\mu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)$$

$$-\frac{\partial^2 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)$$

$$-\frac{\partial A^{\nu}}{\partial x^{\nu}}\frac{1}{2}g^{\nu\mu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)$$

$$-\frac{\partial A^{\nu}}{\partial x^{\nu}}\frac{1}{2}g^{\nu\mu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)$$

$$-A^{\nu}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\mu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)$$

$$+\frac{\partial A^{\mu}}{\partial x^{\nu}}\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)$$

$$+A^{\nu}\frac{1}{2}g^{\nu\mu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)\frac{1}{2}g^{\nu\nu}\left(\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}+\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}-\frac{\partial g_{\nu\nu}}{\partial x^{\nu}}\right)(92)$$

$$=\frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}$$

$$+\frac{\partial^2}{\partial x^{\nu}\partial x^{\nu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g_{\nu}^{\mu}}{\partial x^{\nu}}\right)\right)\ (93)$$

$$+\frac{\partial}{\partial x^{\nu}}\left(\frac{\partial A^{\nu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g_{\nu}^{\mu}}{\partial x^{\nu}}\right)\right)\ (94)$$

$$-\frac{\partial}{\partial x^{\nu}}\left(\frac{\partial A^{\mu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g_{\nu}^{\nu}}{\partial x^{\nu}}\right)\right)\ (95)$$

$$+\frac{\partial^2 A^{\nu}}{\partial x^{\nu}\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g_{\nu}^{\mu}}{\partial x^{\nu}}\right)\ (96)$$

$$+\frac{\partial}{\partial x^{\nu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g_{\nu}^{\nu}}{\partial x^{\nu}}\right)\right)\frac{1}{2}\left(\frac{\partial g_{\nu}^{\mu}}{\partial x^{\nu}}\right)\ (97)$$

$$-2\frac{\partial A^{\nu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g_{\nu}^{\nu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g_{\nu}^{\mu}}{\partial x^{\nu}}\right)\ (98)$$

$$-2\frac{\partial^2 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g_{\nu}^{\nu}}{\partial x^{\nu}}\right)\ (99)$$

$$-2\frac{\partial}{\partial x^{\nu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g_{\nu}^{\mu}}{\partial x^{\nu}}\right)\right)\frac{1}{2}\left(\frac{\partial g_{\nu}^{\nu}}{\partial x^{\nu}}\right)\ (100)$$

$$+2\frac{\partial A^{\mu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g_{\nu}^{\nu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g_{\nu}^{\nu}}{\partial x^{\nu}}\right) \quad (101)$$

from Definision 8. I rewrite dummy index of (93) in the index which isn't used in this term and must get

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}+\frac{\partial^2}{\partial x^{\nu}\partial x^{\nu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g_{\sigma}^{\nu}}{\partial x^{\nu}}\right)\right)..., \quad (102)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}+\frac{\partial^2}{\partial x^{\nu}\partial x^{\nu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g_{\nu}^{\mu}}{\partial x^{\sigma}}\right)\right)..., \quad (103)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}+\frac{\partial^2}{\partial x^{\sigma}\partial x^{\nu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g_{\nu}^{\mu}}{\partial x^{\nu}}\right)\right)..., \quad (104)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}+\frac{\partial^2}{\partial x^{\nu}\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g_{\nu}^{\mu}}{\partial x^{\nu}}\right)\right).... \quad (105)$$

because (92) must be an equation of the tensor. I get the conclusion that (102), (103), (104), (105) doesn't satisfy Binary Law from Definision 4. Similarly, I rewrite dummy index of (94) in the index which isn't used in this term and must get

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}+\frac{\partial}{\partial x^{\nu}}\left(\frac{\partial A^{\sigma}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g_{\sigma}^{\mu}}{\partial x^{\nu}}\right)\right)..., \quad (106)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}+\frac{\partial}{\partial x^{\nu}}\left(\frac{\partial A^{\sigma}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g_{\nu}^{\mu}}{\partial x^{\sigma}}\right)\right)..., \quad (107)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}+\frac{\partial}{\partial x^{\nu}}\left(\frac{\partial A^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g_{\nu}^{\mu}}{\partial x^{\nu}}\right)\right)..., \quad (108)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}+\frac{\partial}{\partial x^{\sigma}}\left(\frac{\partial A^{\sigma}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g_{\nu}^{\mu}}{\partial x^{\nu}}\right)\right)..., \quad (109)$$

because (92) must be an equation of the tensor. I get the conclusion that (106), (107), (108), (109) doesn't satisfy Binary Law from Definision 4. Similarly, I rewrite dummy index of (95) in the index which isn't used in this term and must get

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}-\frac{\partial}{\partial x^{\nu}}\left(\frac{\partial A^{\mu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g_{\sigma}^{\sigma}}{\partial x^{\nu}}\right)\right)..., \quad (110)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}-\frac{\partial}{\partial x^{\nu}}\left(\frac{\partial A^{\mu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g_{\nu}^{\sigma}}{\partial x^{\sigma}}\right)\right)..., \quad (111)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}-\frac{\partial}{\partial x^{\nu}}\left(\frac{\partial A^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g_{\nu}^{\sigma}}{\partial x^{\nu}}\right)\right)..., \quad (112)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}-\frac{\partial}{\partial x^{\sigma}}\left(\frac{\partial A^{\mu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g_{\nu}^{\sigma}}{\partial x^{\nu}}\right)\right).... \quad (113)$$

because (92) must be an equation of the tensor. I get the conclusion that (110), (111), (112), (113) doesn't satisfy Binary Law from Definision4. Similarly, I rewrite dummy index of (96) in the index which isn't used in this term and must get

$$A^{\mu}_{;\nu;\nu;\nu} = \frac{\partial^3 A^{\mu}}{\partial x^{\nu} \partial x^{\nu} \partial x^{\nu}} + \frac{\partial^2 A^{\sigma}}{\partial x^{\nu} \partial x^{\nu}} \frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\nu}}\right) \ldots, \quad (114)$$

$$A^{\mu}_{;\nu;\nu;\nu} = \frac{\partial^3 A^{\mu}}{\partial x^{\nu} \partial x^{\nu} \partial x^{\nu}} + \frac{\partial^2 A^{\sigma}}{\partial x^{\nu} \partial x^{\nu}} \frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\sigma}}\right) \ldots, \quad (115)$$

$$A^{\mu}_{;\nu;\nu;\nu} = \frac{\partial^3 A^{\mu}}{\partial x^{\nu} \partial x^{\nu} \partial x^{\nu}} + \frac{\partial^2 A^{\sigma}}{\partial x^{\sigma} \partial x^{\nu}} \frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\right) \ldots, \quad (116)$$

$$A^{\mu}_{;\nu;\nu;\nu} = \frac{\partial^3 A^{\mu}}{\partial x^{\nu} \partial x^{\nu} \partial x^{\nu}} + \frac{\partial^2 A^{\sigma}}{\partial x^{\nu} \partial x^{\sigma}} \frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\right) \ldots. \quad (117)$$

because (92) must be an equation of the tensor. I get the conclusion that (114), (115), (116), (117) doesn’t satisfy Binary Law from Definision 4. Similarly, I rewrite dummy index of (97) in the index which isn't used in this term and must get

$$A^{\mu}_{;\mu;\mu;\mu} = \frac{\partial^3 A^{\mu}}{\partial x^{\nu} \partial x^{\nu} \partial x^{\nu}} + \frac{\partial}{\partial x^{\nu}}\left(A^{\sigma} \frac{1}{2}\left(\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\nu}}\right)\right) \frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\right) \ldots, \quad (118)$$

$$A^{\mu}_{;\mu;\mu;\mu} = \frac{\partial^3 A^{\mu}}{\partial x^{\nu} \partial x^{\nu} \partial x^{\nu}} + \frac{\partial}{\partial x^{\nu}}\left(A^{\sigma} \frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\nu}}\right)\right) \frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\nu}}\right) \ldots, \quad (119)$$

$$A^{\mu}_{;\mu;\mu;\mu} = \frac{\partial^3 A^{\mu}}{\partial x^{\nu} \partial x^{\nu} \partial x^{\nu}} + \frac{\partial}{\partial x^{\nu}}\left(A^{\sigma} \frac{1}{2}\left(\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\sigma}}\right)\right) \frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\nu}}\right) \ldots, \quad (120)$$

$$A^{\mu}_{;\mu;\mu;\mu} = \frac{\partial^3 A^{\mu}}{\partial x^{\nu} \partial x^{\nu} \partial x^{\nu}} + \frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma} \frac{1}{2}\left(\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\nu}}\right)\right) \frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\nu}}\right) \ldots, \quad (121)$$

$$A^{\mu}_{;\mu;\mu;\mu} = \frac{\partial^3 A^{\mu}}{\partial x^{\nu} \partial x^{\nu} \partial x^{\nu}} + \frac{\partial}{\partial x^{\nu}}\left(A^{\sigma} \frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\nu}}\right)\right) \frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\sigma}}\right) \ldots, \quad (122)$$

$$A^{\mu}_{;\mu;\mu;\mu} = \frac{\partial^3 A^{\mu}}{\partial x^{\nu} \partial x^{\nu} \partial x^{\nu}} + \frac{\partial}{\partial x^{\nu}}\left(A^{\sigma} \frac{1}{2}\left(\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\sigma}}\right)\right) \frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\sigma}}\right) \ldots, \quad (123)$$

$$A^{\mu}_{;\mu;\mu;\mu} = \frac{\partial^3 A^{\mu}}{\partial x^{\nu} \partial x^{\nu} \partial x^{\nu}} + \frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma} \frac{1}{2}\left(\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\nu}}\right)\right) \frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\sigma}}\right) \ldots, \quad (124)$$

$$A^{\mu}_{;\mu;\mu;\mu} = \frac{\partial^3 A^{\mu}}{\partial x^{\nu} \partial x^{\nu} \partial x^{\nu}} + \frac{\partial}{\partial x^{\nu}}\left(A^{\sigma} \frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\sigma}}\right)\right) \frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\right) \ldots, \quad (125)$$

$$A^{\mu}_{;\mu;\mu;\mu} = \frac{\partial^3 A^{\mu}}{\partial x^{\nu} \partial x^{\nu} \partial x^{\nu}} + \frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma} \frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\nu}}\right)\right) \frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\right) \ldots, \quad (126)$$

$$A^{\mu}_{;\mu;\mu;\mu} = \frac{\partial^3 A^{\mu}}{\partial x^{\nu} \partial x^{\nu} \partial x^{\nu}} + \frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma} \frac{1}{2}\left(\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\sigma}}\right)\right) \frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\right) \ldots. \quad (127)$$

because (92) must be an equation of the tensor. I get the conclusion that (118) ~ (127) doesn’t satisfy Binary Law from Definision 4. Similarly, I rewrite dummy index of (98) in the index which isn’t used in this term and must get

$$A^{\mu}_{;\nu;\nu;\nu} = \frac{\partial^3 A^{\mu}}{\partial x^{\nu} \partial x^{\nu} \partial x^{\nu}} - 2\frac{\partial A^{\sigma}}{\partial x^{\nu}} \frac{1}{2}\left(\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\nu}}\right) \frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\right) \ldots, \quad (128)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}-2\frac{\partial A^{\sigma}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\nu}}\right)..., \quad (129)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}-2\frac{\partial A^{\sigma}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\nu}}\right)..., \quad (130)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}-2\frac{\partial A^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\nu}}\right)..., \quad (131)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}-2\frac{\partial A^{\sigma}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\sigma}}\right)..., \quad (132)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}-2\frac{\partial A^{\sigma}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\sigma}}\right)..., \quad (133)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}-2\frac{\partial A^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\sigma}}\right)..., \quad (134)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}-2\frac{\partial A^{\sigma}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\right)..., \quad (135)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}-2\frac{\partial A^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\right)..., \quad (136)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}-2\frac{\partial A^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\right).... \quad (137)$$

because (92) must be an equation of the tensor. I get the conclusion that (128) ~ (137) doesn't satisfy Binary Law from Definision 4. Similarly, I rewrite dummy index of (99) in the index which isn't used in this term and must get

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}-2\frac{\partial^2 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\nu}}\right)..., \quad (138)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}-2\frac{\partial^2 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\sigma}}\right)..., \quad (139)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}-2\frac{\partial^2 A^{\mu}}{\partial x^{\sigma}\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\nu}}\right)..., \quad (140)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}-2\frac{\partial^2 A^{\mu}}{\partial x^{\nu}\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\nu}}\right).... \quad (141)$$

because (92) must be an equation of the tensor. I get the conclusion that (138), (139), (140), (141) doesn't satisfy Binary Law from Definision 4. Similarly, I rewrite dummy index of (100) in the index which isn't used in this term and must get

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}-2\frac{\partial}{\partial x^{\nu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\sigma}}\right)..., \quad (142)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}-2\frac{\partial}{\partial x^{\nu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\nu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\nu}}\right)..., \quad (143)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu}-2\frac{\partial}{\partial x^\nu}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^\mu_\nu}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\nu}\right)..., \quad (144)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu}-2\frac{\partial}{\partial x^\sigma}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^\mu_\nu}{\partial x^\nu}\right)\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\nu}\right)..., \quad (145)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu}-2\frac{\partial}{\partial x^\nu}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x^\nu}\right)\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\nu}{\partial x^\sigma}\right)..., \quad (146)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu}-2\frac{\partial}{\partial x^\nu}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^\mu_\nu}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\nu}{\partial x^\sigma}\right)..., \quad (147)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu}-2\frac{\partial}{\partial x^\sigma}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^\mu_\nu}{\partial x^\nu}\right)\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\nu}{\partial x^\sigma}\right)..., \quad (148)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu}-2\frac{\partial}{\partial x^\nu}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\nu}{\partial x^\nu}\right)..., \quad (149)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu}-2\frac{\partial}{\partial x^\sigma}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x^\nu}\right)\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\nu}{\partial x^\nu}\right)..., \quad (150)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu}-2\frac{\partial}{\partial x^\sigma}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^\mu_\nu}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\nu}{\partial x^\nu}\right).... \quad (151)$$

because (92) must be an equation of the tensor. I get the conclusion that (142) ~ (151) doesn't satisfy Binary Law from Definision 4. Similarly, I rewrite dummy index of (101) in the index which isn't used in this term and must get

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu}+2\frac{\partial A^\mu}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g^\sigma_\nu}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\sigma}\right)..., \quad (152)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu}+2\frac{\partial A^\mu}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\nu}\right)..., \quad (153)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu}+2\frac{\partial A^\mu}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g^\sigma_\nu}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\nu}\right)..., \quad (154)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu}+2\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^\sigma_\nu}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\nu}\right)..., \quad (155)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu}+2\frac{\partial A^\mu}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\nu}{\partial x^\sigma}\right)..., \quad (156)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu}+2\frac{\partial A^\mu}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g^\sigma_\nu}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\nu}{\partial x^\sigma}\right)..., \quad (157)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu}+2\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^\sigma_\nu}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\nu}{\partial x^\sigma}\right)..., \quad (158)$$

$$A^{\mu}_{;\nu;\nu;\nu}=\frac{\partial^3 A^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu}+2\frac{\partial A^\mu}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\nu}{\partial x^\nu}\right)..., \quad (159)$$

$$A^{\mu}_{;\nu;\nu;\nu} = \frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}} + 2\frac{\partial A^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\nu}}\right)..., \quad (160)$$

$$A^{\mu}_{;\nu;\nu;\nu} = \frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}} + 2\frac{\partial A^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\nu}}{\partial x^{\nu}}\right).... \quad (161)$$

because (92) must be an equation of the tensor. I get the conclusion that (152) ~ (161) doesn't satisfy Binary Law from Definision 4. If (102) ~ (161) satisfies Binary Law, Definision 8 is an equation of the tensor satisfying Binary Law here. However, I get the conclusion that Definision 8 isn't an equation of the tensor satisfying Binary Law because (102) ~ (161) doesn't satisfy Binary Law.

– End Proof –

Proposition 6 $A^{\mu}_{;\nu;\nu;\nu} = \frac{\partial^3 A^{\mu}}{\partial x^{\nu}\partial x^{\nu}\partial x^{\nu}}$ is the equation that was rewritten so that Definision8 is an equation of the tensor satisfying Binary Law.

Proof: I rewrite index ν of the coordinate systems of (102), (103), (104), (105) by Definision 3 and get

$$-A^{\mu}_{;\mu;\mu;\mu} = -\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}} - \frac{\partial^2}{\partial x^{\mu}\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)\right)..., \quad (162)$$

$$-A^{\mu}_{;\mu;\mu;\mu} = -\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}} - \frac{\partial^2}{\partial x^{\mu}\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)\right)..., \quad (163)$$

$$-A^{\mu}_{;\mu;\mu;\mu} = -\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}} - \frac{\partial^2}{\partial x^{\sigma}\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)\right)..., \quad (164)$$

$$-A^{\mu}_{;\mu;\mu;\mu} = -\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}} - \frac{\partial^2}{\partial x^{\mu}\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)\right).... \quad (165)$$

Because two same covariant index existed in $-A^{\mu}_{\mu;\mu;\mu}$ of (162), (163), (164), (165), I decided not to handle (162), (163) (164), (165). Furthermore, I rewrite index ν of the coordinate systems of (102), (103), (104), (105) by Definision 2 and get

$$A^{\mu;\mu;\mu;\mu} = \frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}} + \frac{\partial^2}{\partial x_{\mu}\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)\right)..., \quad (166)$$

$$A^{\mu;\mu;\mu;\mu} = \frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}} + \frac{\partial^2}{\partial x_{\mu}\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)\right)..., \quad (167)$$

$$A^{\mu;\mu;\mu;\mu} = \frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}} + \frac{\partial^2}{\partial x^{\sigma}\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x_{\mu}}\right)\right)..., \quad (168)$$

$$A^{\mu;\mu;\mu;\mu} = \frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}} + \frac{\partial^2}{\partial x_{\mu}\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x_{\mu}}\right)\right).... \quad (169)$$

Because two same contravariant index existed in $A^{\mu;\mu;\mu;\mu}$ of (166), (167), (168), (169), I decided not to handle (166), (167), (168), (169). Furthermore, I rewrite index ν of the coordinate systems of (102), (103), (104), (105) by Definision 3, Definision 2 and get

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial^2}{\partial x^{\mu}\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)\right)\ldots,\quad(170)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial^2}{\partial x^{\mu}\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)\right)\ldots,\quad(171)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial^2}{\partial x^{\sigma}\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)\right)\ldots,\quad(172)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial^2}{\partial x_{\mu}\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)\right)\ldots.\quad(173)$$

There isn't index ν in the second term of the right side of (170), (171), (172), (173) at all here. Therefore, I rewrite dummy index σ of (170), (171), (172), (173) in dummy index ν and can get

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial^2}{\partial x^{\mu}\partial x_{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\right)\right)\ldots,\quad(174)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial^2}{\partial x^{\mu}\partial x_{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}\right)\right)\ldots,\quad(175)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial^2}{\partial x^{\nu}\partial x_{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)\right)\ldots,\quad(176)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial^2}{\partial x_{\mu}\partial x^{\nu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)\right)\ldots.\quad(177)$$

Furthermore, I rewrite index ν of the coordinate systems of (102), (103), (104), (105) by Definision 3, Definision 2 and get

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial^2}{\partial x_{\mu}\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)\right)\ldots,\quad(178)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial^2}{\partial x_{\mu}\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)\right)\ldots,\quad(179)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial^2}{\partial x^{\sigma}\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)\right)\ldots,\quad(180)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial^2}{\partial x^{\mu}\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)\right)\ldots.\quad(181)$$

There isn't index ν in the second term of the right side of (178), (179), (180), (181) at all here. Therefore, I rewrite dummy index σ of (178), (179), (180), (181) in dummy index ν and can get

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial^2}{\partial x_{\mu}\partial x^{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\right)\right)\ldots,\quad(182)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial^2}{\partial x_{\mu}\partial x^{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}\right)\right)\ldots,\quad(183)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial^2}{\partial x^{\nu}\partial x^{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)\right)..., \quad (184)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial^2}{\partial x^{\mu}\partial x^{\nu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)\right).... \quad (185)$$

Furthermore, I rewrite index ν of the coordinate systems of (102), (103), (104), (105) by Definision 3, Definision 2 and get

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}}+\frac{\partial^2}{\partial x^{\mu}\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)\right)..., \quad (186)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}}+\frac{\partial^2}{\partial x^{\mu}\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)\right)..., \quad (187)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}}+\frac{\partial^2}{\partial x^{\sigma}\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\mu}}\right)\right)..., \quad (188)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}}+\frac{\partial^2}{\partial x^{\mu}\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\mu}}\right)\right).... \quad (189)$$

There isn't index ν in the second term of the right side of (186), (187), (188), (189) at all here. Therefore, I rewrite dummy index σ of (186), (187), (188), (189) in dummy index ν and can get

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}}+\frac{\partial^2}{\partial x^{\mu}\partial x^{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x_{\mu}}\right)\right)..., \quad (190)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}}+\frac{\partial^2}{\partial x^{\mu}\partial x^{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\nu}}\right)\right)..., \quad (191)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}}+\frac{\partial^2}{\partial x^{\nu}\partial x^{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\mu}}\right)\right)..., \quad (192)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}}+\frac{\partial^2}{\partial x^{\mu}\partial x^{\nu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\mu}}\right)\right).... \quad (193)$$

By the way, by establishment of $A^{\mu}_{;\dot{\nu};\ddot{\nu};\hat{\nu}}=A^{\mu}_{;\dot{\nu};\hat{\nu};\ddot{\nu}}=A^{\mu}_{;\hat{\nu};\dot{\nu};\ddot{\nu}}=A^{\mu}_{;\ddot{\nu};\dot{\nu};\hat{\nu}}=A^{\mu}_{;\ddot{\nu};\hat{\nu};\dot{\nu}}=A^{\mu}_{;\hat{\nu};\ddot{\nu};\dot{\nu}}$, (174), (182), (190) must be equal each other. (174), (182), (190) are equal each other in consideration of Definision 5. Furthermore, by establishment of $A^{\mu}_{;\dot{\nu};\ddot{\nu};\hat{\nu}}=A^{\mu}_{;\dot{\nu};\hat{\nu};\ddot{\nu}}=A^{\mu}_{;\hat{\nu};\dot{\nu};\ddot{\nu}}=A^{\mu}_{;\ddot{\nu};\dot{\nu};\hat{\nu}}=A^{\mu}_{;\ddot{\nu};\hat{\nu};\dot{\nu}}=A^{\mu}_{;\hat{\nu};\ddot{\nu};\dot{\nu}}$, (175), (183), (191) must be equal each other. (175), (183), (191) aren't equal each other in consideration of Definision 5. Thus, I decide not to handle (175), (183), (191). Furthermore, by establishment of $A^{\mu}_{;\dot{\nu};\ddot{\nu};\hat{\nu}}=A^{\mu}_{;\dot{\nu};\hat{\nu};\ddot{\nu}}=A^{\mu}_{;\hat{\nu};\dot{\nu};\ddot{\nu}}=A^{\mu}_{;\ddot{\nu};\dot{\nu};\hat{\nu}}=A^{\mu}_{;\ddot{\nu};\hat{\nu};\dot{\nu}}=A^{\mu}_{;\hat{\nu};\ddot{\nu};\dot{\nu}}$, (176), (184), (192) must be equal each other. (176), (184), (192) aren't equal each other in consideration of Definision5. Thus, I decide not to handle (176), (184), (192). Furthermore, by establishment of $A^{\mu}_{;\dot{\nu};\ddot{\nu};\hat{\nu}}=A^{\mu}_{;\dot{\nu};\hat{\nu};\ddot{\nu}}=A^{\mu}_{;\hat{\nu};\dot{\nu};\ddot{\nu}}=A^{\mu}_{;\ddot{\nu};\dot{\nu};\hat{\nu}}=A^{\mu}_{;\ddot{\nu};\hat{\nu};\dot{\nu}}=A^{\mu}_{;\hat{\nu};\ddot{\nu};\dot{\nu}}$, (177), (185), (193) must be equal each other. (177), (185), (193) aren't equal each other in consideration of Definision 5. Thus, I decide not to handle (177), (185), (193). Furthermore, I rewrite index ν of the coordinate systems of (102), (103), (104), (105) by Definision 3, Definision 2 and get

$$-(A^{\mu}_{;\mu})^{;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x_{\mu}}-\frac{\partial^2}{\partial x_{\mu}\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)\right)..., \quad (194)$$

$$-(A^{\mu}_{;\mu})^{;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x_{\mu}}-\frac{\partial^2}{\partial x_{\mu}\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)\right)\ldots,\quad(195)$$

$$-(A^{\mu}_{;\mu})^{;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x_{\mu}}-\frac{\partial^2}{\partial x^{\sigma}\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)\right)\ldots,\quad(196)$$

$$-(A^{\mu}_{;\mu})^{;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x_{\mu}}-\frac{\partial^2}{\partial x_{\mu}\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)\right)\ldots.\quad(197)$$

Because two same contravariant index existed in $-(A^{\mu}_{;\mu})^{;\mu;\mu}$ of (194), (195), (196), (197), I decided not to handle (194), (195), (196), (197). Furthermore, I rewrite index ν of the coordinate systems of (102), (103), (104), (105) by Definision 3, Definision 2 and get

$$-\left(A^{\mu;\mu;\mu}\right)_{;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x^{\mu}}-\frac{\partial^2}{\partial x_{\mu}\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)\right)\ldots,\quad(198)$$

$$-\left(A^{\mu;\mu;\mu}\right)_{;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x^{\mu}}-\frac{\partial^2}{\partial x_{\mu}\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)\right)\ldots,\quad(199)$$

$$-\left(A^{\mu;\mu;\mu}\right)_{;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x^{\mu}}-\frac{\partial^2}{\partial x^{\sigma}\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x_{\mu}}\right)\right)\ldots,\quad(200)$$

$$-\left(A^{\mu;\mu;\mu}\right)_{;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x^{\mu}}-\frac{\partial^2}{\partial x^{\mu}\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x_{\mu}}\right)\right)\ldots.\quad(201)$$

Because two same contravariant index existed in $-(A^{\mu;\mu;\mu})_{;\mu}$ of (198), (199), (200), (201), I decided not to handle (198), (199), (200), (201). Furthermore, I rewrite index ν of the coordinate systems of (102), (103), (104), (105) by Definision 3, Definision 2 and get

$$-(A^{\mu;\mu})^{;\mu}_{;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x_{\mu}}-\frac{\partial^2}{\partial x^{\mu}\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)\right)\ldots,\quad(202)$$

$$-(A^{\mu;\mu})^{;\mu}_{;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x_{\mu}}-\frac{\partial^2}{\partial x^{\mu}\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)\right)\ldots,\quad(203)$$

$$-(A^{\mu;\mu})^{;\mu}_{;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x_{\mu}}-\frac{\partial^2}{\partial x^{\sigma}\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\mu}}\right)\right)\ldots,\quad(204)$$

$$-(A^{\mu;\mu})^{;\mu}_{;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x_{\mu}}-\frac{\partial^2}{\partial x_{\mu}\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\mu}}\right)\right)\ldots.\quad(205)$$

Because two same contravariant index existed in $-(A^{\mu;\mu})^{;\mu}_{;\mu}$ of (202), (203), (204), (205), I decided not to handle (202), (203), (204), (205). Similarly, I rewrite index ν of the coordinate systems of (106), (107), (108), (109) by Definision 3 and get

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}-\frac{\partial}{\partial x^{\mu}}\left(\frac{\partial A^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)\right)\ldots,\quad(206)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}-\frac{\partial}{\partial x^{\mu}}\left(\frac{\partial A^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)\right)\ldots,\quad(207)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}-\frac{\partial}{\partial x^{\mu}}\left(\frac{\partial A^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)\right)\ldots,\quad(208)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}-\frac{\partial}{\partial x^{\sigma}}\left(\frac{\partial A^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)\right)\ldots.\quad(209)$$

Because two same covariant index existed in $-A^{\mu}_{;\mu;\mu;\mu}$ of (206), (207), (208), (209), I decided not to handle (206), (207), (208), (209). Furthermore, I rewrite index ν of the coordinate systems of (106), (107), (108), (109) by Definision 2 and get

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x_{\mu}}\left(\frac{\partial A^{\sigma}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)\right)\ldots,\quad(210)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x_{\mu}}\left(\frac{\partial A^{\sigma}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)\right)\ldots,\quad(211)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x_{\mu}}\left(\frac{\partial A^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x_{\mu}}\right)\right)\ldots,\quad(212)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x^{\sigma}}\left(\frac{\partial A^{\sigma}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x_{\mu}}\right)\right)\ldots.\quad(213)$$

Because two same contravariant index existed in $A^{\mu;\mu;\mu;\mu}$ of (210), (211), (212), (213), I decided not to handle (210), (211), (212), (213). Furthermore, I rewrite index ν of the coordinate systems of (106), (107), (108), (109) by Definision 3, Definision 2 and get

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x_{\mu}}\left(\frac{\partial A^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)\right)\ldots,\quad(214)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x_{\mu}}\left(\frac{\partial A^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)\right)\ldots,\quad(215)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x_{\mu}}\left(\frac{\partial A^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)\right)\ldots,\quad(216)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x^{\sigma}}\left(\frac{\partial A^{\sigma}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)\right)\ldots.\quad(217)$$

There isn't index ν in the second term of the right side of (214), (215), (216), (217) at all here. Therefore, I rewrite dummy index σ of (214), (215), (216), (217) in dummy index ν and can get

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x_{\mu}}\left(\frac{\partial A^{\nu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\right)\right)\ldots,\quad(218)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x_{\mu}}\left(\frac{\partial A^{\nu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}\right)\right)\ldots,\quad(219)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x_{\mu}}\left(\frac{\partial A^{\nu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)\right)\ldots,\quad (220)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x^{\nu}}\left(\frac{\partial A^{\nu}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)\right)\ldots.\quad (221)$$

Furthermore, I rewrite index ν of the coordinate systems of (106), (107), (108), (109) by Definision 3, Definision 2 and get

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(\frac{\partial A^{\sigma}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)\right)\ldots,\quad (222)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(\frac{\partial A^{\sigma}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)\right)\ldots,\quad (223)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(\frac{\partial A^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)\right)\ldots,\quad (224)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\sigma}}\left(\frac{\partial A^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)\right)\ldots.\quad (225)$$

There isn't index ν in the second term of the right side of (222), (223), (224), (225) at all here. Therefore, I rewrite dummy index σ of (222), (223), (224), (225) in dummy index ν and can get

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(\frac{\partial A^{\nu}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\right)\right)\ldots,\quad (226)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(\frac{\partial A^{\nu}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}\right)\right)\ldots,\quad (227)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(\frac{\partial A^{\nu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)\right)\ldots,\quad (228)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\nu}}\left(\frac{\partial A^{\nu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)\right)\ldots.\quad (229)$$

Furthermore, I rewrite index ν of the coordinate systems of (106), (107), (108), (109) by Definision 3, Definision 2 and get

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(\frac{\partial A^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)\right)\ldots,\quad (230)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(\frac{\partial A^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)\right)\ldots,\quad (231)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(\frac{\partial A^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\mu}}\right)\right)\ldots,\quad (232)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}+\frac{\partial}{\partial x^\sigma}\left(\frac{\partial A^\sigma}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\mu}\right)\right).... \quad (233)$$

There isn't index ν in the second term of the right side of (230), (231), (232), (233) at all here. Therefore, I rewrite dummy index σ of (230), (231), (232), (233) in dummy index ν and can get

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}+\frac{\partial}{\partial x^\mu}\left(\frac{\partial A^\nu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^\mu_\nu}{\partial x_\mu}\right)\right)..., \quad (234)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}+\frac{\partial}{\partial x^\mu}\left(\frac{\partial A^\nu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\nu}\right)\right)..., \quad (235)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}+\frac{\partial}{\partial x^\mu}\left(\frac{\partial A^\nu}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\mu}\right)\right)..., \quad (236)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}+\frac{\partial}{\partial x^\nu}\left(\frac{\partial A^\nu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\mu}\right)\right).... \quad (237)$$

By the way, by establishment of $A^\mu_{;\dot\nu;\ddot\nu;\hat\nu}=A^\mu_{;\dot\nu;\hat\nu;\ddot\nu}=A^\mu_{;\hat\nu;\dot\nu;\ddot\nu}=A^\mu_{;\ddot\nu;\dot\nu;\hat\nu}=A^\mu_{;\ddot\nu;\hat\nu;\dot\nu}=A^\mu_{;\hat\nu;\ddot\nu;\dot\nu}$, (218), (226), (234) must be equal each other. (218), (226), (234) are equal each other in consideration of Definision 5. Furthermore, by establishment of $A^\mu_{;\dot\nu;\ddot\nu;\hat\nu}=A^\mu_{;\dot\nu;\hat\nu;\ddot\nu}=A^\mu_{;\hat\nu;\dot\nu;\ddot\nu}=A^\mu_{;\ddot\nu;\dot\nu;\hat\nu}=A^\mu_{;\ddot\nu;\hat\nu;\dot\nu}=A^\mu_{;\hat\nu;\ddot\nu;\dot\nu}$, (219), (227), (235) must be equal each other. (219), (227), (235) aren't equal each other in consideration of Definision5. Thus, I decide not to handle (219), (227), (235). Furthermore, by establishment of $A^\mu_{;\dot\nu;\ddot\nu;\hat\nu}=A^\mu_{;\dot\nu;\hat\nu;\ddot\nu}=A^\mu_{;\hat\nu;\dot\nu;\ddot\nu}=A^\mu_{;\ddot\nu;\dot\nu;\hat\nu}=A^\mu_{;\ddot\nu;\hat\nu;\dot\nu}=A^\mu_{;\hat\nu;\ddot\nu;\dot\nu}$, (220), (228), (236) must be equal each other. (220), (228), (236) aren't equal each other in consideration of Definision 5. Thus, I decide not to handle (220), (228), (236). Furthermore, by establishment of $A^\mu_{;\dot\nu;\ddot\nu;\hat\nu}=A^\mu_{;\dot\nu;\hat\nu;\ddot\nu}=A^\mu_{;\hat\nu;\dot\nu;\ddot\nu}=A^\mu_{;\ddot\nu;\dot\nu;\hat\nu}=A^\mu_{;\ddot\nu;\hat\nu;\dot\nu}=A^\mu_{;\hat\nu;\ddot\nu;\dot\nu}$, (221), (229), (237) must be equal each other. (221), (229), (237) aren't equal each other in consideration of Definision 5. Thus, I decide not to handle (221), (229), (237). Furthermore, I rewrite index ν of the coordinate systems of (106), (107), (108), (109) by Definision 3, Definision 2 and get

$$-\left(A^\mu_{;\mu}\right)^{;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu}-\frac{\partial}{\partial x_\mu}\left(\frac{\partial A^\sigma}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x^\mu}\right)\right)..., \quad (238)$$

$$-\left(A^\mu_{;\mu}\right)^{;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu}-\frac{\partial}{\partial x_\mu}\left(\frac{\partial A^\sigma}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^\mu_\mu}{\partial x^\sigma}\right)\right)..., \quad (239)$$

$$-\left(A^\mu_{;\mu}\right)^{;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu}-\frac{\partial}{\partial x_\mu}\left(\frac{\partial A^\sigma}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^\mu_\mu}{\partial x_\mu}\right)\right)..., \quad (240)$$

$$-\left(A^\mu_{;\mu}\right)^{;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu}-\frac{\partial}{\partial x^\sigma}\left(\frac{\partial A^\sigma}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^\mu_\mu}{\partial x_\mu}\right)\right).... \quad (241)$$

Because two same contravariant index existed in $-\left(A^\mu_{;\mu}\right)^{;\mu;\mu}$ of (238), (239), (240), (241), I decided not to handle (238), (239), (240), (241). Furthermore, I rewrite index ν of the coordinate systems of (106), (107), (108), (109) by Definision 3, Definision 2 and get

$$-\left(A^{\mu;\mu;\mu;}\right)_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}-\frac{\partial}{\partial x^\mu}\left(\frac{\partial A^\sigma}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x_\mu}\right)\right)..., \quad (242)$$

$$-\left(A^{\mu;\mu;\mu;}\right)_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}-\frac{\partial}{\partial x^\mu}\left(\frac{\partial A^\sigma}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\sigma}\right)\right)\ldots,\quad (243)$$

$$-\left(A^{\mu;\mu;\mu;}\right)_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}-\frac{\partial}{\partial x^\mu}\left(\frac{\partial A^\sigma}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x_\mu}\right)\right)\ldots,\quad (244)$$

$$-\left(A^{\mu;\mu;\mu;}\right)_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}-\frac{\partial}{\partial x^\sigma}\left(\frac{\partial A^\sigma}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x_\mu}\right)\right)\ldots.\quad (245)$$

Because two same contravariant index existed in $-(A^{\mu;\mu;\mu})_{;\mu}$ of (242), (243), (244), (245), I decided not to handle (242), (243), (244), (245). Furthermore, I rewrite index ν of the coordinate systems of (106), (107), (108), (109) by Definision 3,Definision 2 and get

$$-\left(A^{\mu;\mu;}\right)_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}-\frac{\partial}{\partial x_\mu}\left(\frac{\partial A^\sigma}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_\mu}\right)\right)\ldots,\quad (246)$$

$$-\left(A^{\mu;\mu;}\right)_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}-\frac{\partial}{\partial x_\mu}\left(\frac{\partial A^\sigma}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\sigma}\right)\right)\ldots,\quad (247)$$

$$-\left(A^{\mu;\mu;}\right)_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}-\frac{\partial}{\partial x_\mu}\left(\frac{\partial A^\sigma}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\mu}\right)\right)\ldots,\quad (248)$$

$$-\left(A^{\mu;\mu;}\right)_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}-\frac{\partial}{\partial x^\sigma}\left(\frac{\partial A^\sigma}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\mu}\right)\right)\ldots.\quad (249)$$

Because two same contravariant index existed in $-\left(A^{\mu;\mu}\right)_{;\mu}^{;\mu}$ of (246), (247), (248), (249), I decided not to handle (246), (247), (248), (249). Similarly, I rewrite index ν of the coordinate systems of (110), (111), (112), (113) by Definision 3 and get

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x^\mu}+\frac{\partial}{\partial x^\mu}\left(\frac{\partial A^\mu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^\mu}\right)\right)\ldots,\quad (250)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x^\mu}+\frac{\partial}{\partial x^\mu}\left(\frac{\partial A^\mu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^\sigma}\right)\right)\ldots,\quad (251)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x^\mu}+\frac{\partial}{\partial x^\mu}\left(\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^\mu}\right)\right)\ldots,\quad (252)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x^\mu}+\frac{\partial}{\partial x^\sigma}\left(\frac{\partial A^\mu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^\mu}\right)\right)\ldots.\quad (253)$$

Because two same covariant index existed in $-A^{\mu}_{;\mu;\mu;\mu}$ of (250), (251), (252), (253), I decided not to handle (250), (251), (252), (253). Furthermore, I rewrite index ν of the coordinate systems of (110), (111), (112), (113) by Definision 2 and get

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x_\mu}-\frac{\partial}{\partial x_\mu}\left(\frac{\partial A^\mu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_\mu}\right)\right)\ldots,\quad (254)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x_\mu}-\frac{\partial}{\partial x_\mu}\left(\frac{\partial A^\mu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)\right)\ldots,\quad (255)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x_\mu}-\frac{\partial}{\partial x_\mu}\left(\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_\mu}\right)\right)..., \quad (256)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x_\mu}-\frac{\partial}{\partial x^\sigma}\left(\frac{\partial A^\mu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_\mu}\right)\right).... \quad (257)$$

Because two same contravariant index existed in $A^{\mu;\mu;\mu;\mu}$ of (254), (255), (256), (257), I decided not to handle (254), (255), (256), (257). Furthermore, I rewrite index ν of the coordinate systems of (110), (111), (112), (113) by Definision 3, Definision 2 and get

$$\left(A^\mu_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x_\mu}-\frac{\partial}{\partial x_\mu}\left(\frac{\partial A^\mu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\mu}\right)\right)..., \quad (258)$$

$$\left(A^\mu_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x_\mu}-\frac{\partial}{\partial x_\mu}\left(\frac{\partial A^\mu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^\sigma}\right)\right)..., \quad (259)$$

$$\left(A^\mu_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x_\mu}-\frac{\partial}{\partial x_\mu}\left(\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^\mu}\right)\right)..., \quad (260)$$

$$\left(A^\mu_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x_\mu}-\frac{\partial}{\partial x^\sigma}\left(\frac{\partial A^\mu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^\mu}\right)\right).... \quad (261)$$

There isn't index ν in the second term of the right side of (258), (259), (260), (261) at all here. Therefore, I rewrite dummy index σ of (258), (259), (260), (261) in dummy index ν and can get

$$\left(A^\mu_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x_\mu}-\frac{\partial}{\partial x_\mu}\left(\frac{\partial A^\mu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^\nu_\nu}{\partial x^\mu}\right)\right)..., \quad (262)$$

$$\left(A^\mu_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x_\mu}-\frac{\partial}{\partial x_\mu}\left(\frac{\partial A^\mu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^\nu_\mu}{\partial x^\nu}\right)\right)..., \quad (263)$$

$$\left(A^\mu_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x_\mu}-\frac{\partial}{\partial x_\mu}\left(\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^\mu}\right)\right)..., \quad (264)$$

$$\left(A^\mu_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x_\mu}-\frac{\partial}{\partial x^\sigma}\left(\frac{\partial A^\mu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^\mu}\right)\right).... \quad (265)$$

Furthermore, I rewrite index ν of the coordinate systems of (110), (111), (112), (113) by Definision 3, Definision 2 and get

$$\left(A^\mu_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x^\mu}-\frac{\partial}{\partial x^\mu}\left(\frac{\partial A^\mu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\mu}\right)\right)..., \quad (266)$$

$$\left(A^\mu_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x^\mu}-\frac{\partial}{\partial x^\mu}\left(\frac{\partial A^\mu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^\sigma}\right)\right)..., \quad (267)$$

$$\left(A^\mu_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x^\mu}-\frac{\partial}{\partial x^\mu}\left(\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x_\mu}\right)\right)..., \quad (268)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu} = \frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}} - \frac{\partial}{\partial x^{\sigma}}\left(\frac{\partial A^{\mu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x_{\mu}}\right)\right) .... \quad (269)$$

There isn't index ν in the second term of the right side of (266), (267), (268), (269) at all here. Therefore, I rewrite dummy index σ of (266), (267), (268), (269) in dummy index ν and can get

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu} = \frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}} - \frac{\partial}{\partial x^{\mu}}\left(\frac{\partial A^{\mu}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\right)\right) ..., \quad (270)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu} = \frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}} - \frac{\partial}{\partial x^{\mu}}\left(\frac{\partial A^{\mu}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\right)\right) ..., \quad (271)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu} = \frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}} - \frac{\partial}{\partial x^{\mu}}\left(\frac{\partial A^{\mu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x_{\mu}}\right)\right) ..., \quad (272)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu} = \frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}} - \frac{\partial}{\partial x^{\nu}}\left(\frac{\partial A^{\mu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x_{\mu}}\right)\right) .... \quad (273)$$

Furthermore, I rewrite index ν of the coordinate systems of (110), (111), (112), (113) by Definision 3, Definision 2 and get

$$\left(A^{;\mu;\mu}\right)_{;\mu;\mu} = \frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}} - \frac{\partial}{\partial x^{\mu}}\left(\frac{\partial A^{\mu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_{\mu}}\right)\right) ..., \quad (274)$$

$$\left(A^{;\mu;\mu}\right)_{;\mu;\mu} = \frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}} - \frac{\partial}{\partial x^{\mu}}\left(\frac{\partial A^{\mu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\sigma}}\right)\right) ..., \quad (275)$$

$$\left(A^{;\mu;\mu}\right)_{;\mu;\mu} = \frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}} - \frac{\partial}{\partial x^{\mu}}\left(\frac{\partial A^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\mu}}\right)\right) ..., \quad (276)$$

$$\left(A^{;\mu;\mu}\right)_{;\mu;\mu} = \frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}} - \frac{\partial}{\partial x^{\sigma}}\left(\frac{\partial A^{\mu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\mu}}\right)\right) .... \quad (277)$$

There isn't index ν in the second term of the right side of (274), (275), (276), (277) at all here. Therefore, I rewrite dummy index σ of (274), (275), (276), (277) in dummy index ν and can get

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu} = \frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}} - \frac{\partial}{\partial x^{\mu}}\left(\frac{\partial A^{\mu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x_{\mu}}\right)\right) ..., \quad (278)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu} = \frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}} - \frac{\partial}{\partial x^{\mu}}\left(\frac{\partial A^{\mu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^{\nu}}\right)\right) ..., \quad (279)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu} = \frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}} - \frac{\partial}{\partial x^{\mu}}\left(\frac{\partial A^{\mu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^{\mu}}\right)\right) ..., \quad (280)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu} = \frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}} - \frac{\partial}{\partial x^{\nu}}\left(\frac{\partial A^{\mu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^{\mu}}\right)\right) .... \quad (281)$$

By the way, by establishment of $A^{\mu}_{;\dot{\nu};\ddot{\nu};\hat{\nu}} = A^{\mu}_{;\dot{\nu};\hat{\nu};\ddot{\nu}} = A^{\mu}_{;\hat{\nu};\dot{\nu};\ddot{\nu}} = A^{\mu}_{;\ddot{\nu};\dot{\nu};\hat{\nu}} = A^{\mu}_{;\ddot{\nu};\hat{\nu};\dot{\nu}} = A^{\mu}_{;\hat{\nu};\ddot{\nu};\dot{\nu}}$, (262), (270), (278) must be equal each other. (262), (270), (278) are equal each other in consideration of Definision 5. Furthermore, by establishment of $A^{\mu}_{;\dot{\nu};\ddot{\nu};\hat{\nu}} = A^{\mu}_{;\dot{\nu};\hat{\nu};\ddot{\nu}} = A^{\mu}_{;\hat{\nu};\dot{\nu};\ddot{\nu}} = A^{\mu}_{;\ddot{\nu};\dot{\nu};\hat{\nu}} = A^{\mu}_{;\ddot{\nu};\hat{\nu};\dot{\nu}} = A^{\mu}_{;\hat{\nu};\ddot{\nu};\dot{\nu}}$, (263), (271), (279) must be equal each other. (263), (271), (279) aren't

Citation: Koji Ichidayama (2018) Tensor Satisfying Binary Law about the First, Second, Third-Order Covariant Derivative of the Contravariant Vector. SF J Nuclear Science 2:3.

equal each other in consideration of Definision 5. Thus, I decide not to handle (263), (271), (279). Furthermore, by establishment of $A^{\mu}_{;\dot{\nu};\ddot{\nu};\hat{\nu}} = A^{\mu}_{;\dot{\nu};\hat{\nu};\ddot{\nu}} = A^{\mu}_{;\hat{\nu};\dot{\nu};\ddot{\nu}} = A^{\mu}_{;\ddot{\nu};\dot{\nu};\hat{\nu}} = A^{\mu}_{;\ddot{\nu};\hat{\nu};\dot{\nu}} = A^{\mu}_{;\hat{\nu};\ddot{\nu};\dot{\nu}}$, (264), (272), (280) must be equal each other. (264), (272), (280) aren't equal each other in consideration of Definision5. Thus, I decide not to handle (264), (272), (280). Furthermore, by establishment of $A^{\mu}_{;\dot{\nu};\ddot{\nu};\hat{\nu}} = A^{\mu}_{;\dot{\nu};\hat{\nu};\ddot{\nu}} = A^{\mu}_{;\hat{\nu};\dot{\nu};\ddot{\nu}} = A^{\mu}_{;\ddot{\nu};\dot{\nu};\hat{\nu}} = A^{\mu}_{;\ddot{\nu};\hat{\nu};\dot{\nu}} = A^{\mu}_{;\hat{\nu};\ddot{\nu};\dot{\nu}}$, (265), (273), (281) must be equal each other. (265), (273), (281) aren't equal each other in consideration of Definision 5. Thus, I decide not to handle (265), (273), (281). Furthermore, I rewrite index ν of the coordinate systems of (110), (111), (112), (113) by Definision 3, Definision 2 and get

$$-\left(A^{\mu}_{;\mu}\right)^{;\mu;\mu} = -\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x_{\mu}} + \frac{\partial}{\partial x_{\mu}}\left(\frac{\partial A^{\mu}}{\partial x_{\mu}}\frac{1}{2}\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)..., \quad (282)$$

$$-\left(A^{\mu}_{;\mu}\right)^{;\mu;\mu} = -\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x_{\mu}} + \frac{\partial}{\partial x_{\mu}}\left(\frac{\partial A^{\mu}}{\partial x_{\mu}}\frac{1}{2}\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)..., \quad (283)$$

$$-\left(A^{\mu}_{;\mu}\right)^{;\mu;\mu} = -\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x_{\mu}} + \frac{\partial}{\partial x_{\mu}}\left(\frac{\partial A^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma}_{\mu}}{\partial x_{\mu}}\right)..., \quad (284)$$

$$-\left(A^{\mu}_{;\mu}\right)^{;\mu;\mu} = -\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x_{\mu}} + \frac{\partial}{\partial x^{\sigma}}\left(\frac{\partial A^{\mu}}{\partial x_{\mu}}\frac{1}{2}\frac{\partial g^{\sigma}_{\mu}}{\partial x_{\mu}}\right).... \quad (285)$$

Because two same contravariant index existed in $-\left(A^{\mu}_{;\mu}\right)^{;\mu;\mu}$ of (282), (283), (284), (285), I decided not to handle (282), (283), (284), (285). Furthermore, I rewrite index ν of the coordinate systems of (110), (111), (112), (113) by Definision 3, Definision 2 and get

$$-\left(A^{\mu;\mu;\mu}\right)_{;\mu} = -\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x^{\mu}} + \frac{\partial}{\partial x^{\mu}}\left(\frac{\partial A^{\mu}}{\partial x_{\mu}}\frac{1}{2}\frac{\partial g^{\sigma}_{\sigma}}{\partial x_{\mu}}\right)..., \quad (286)$$

$$-\left(A^{\mu;\mu;\mu}\right)_{;\mu} = -\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x^{\mu}} + \frac{\partial}{\partial x^{\mu}}\left(\frac{\partial A^{\mu}}{\partial x_{\mu}}\frac{1}{2}\frac{\partial g^{\sigma\mu}}{\partial x^{\sigma}}\right)..., \quad (287)$$

$$-\left(A^{\mu;\mu;\mu}\right)_{;\mu} = -\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x^{\mu}} + \frac{\partial}{\partial x^{\mu}}\left(\frac{\partial A^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\frac{\partial g^{\sigma\mu}}{\partial x_{\mu}}\right)..., \quad (288)$$

$$-\left(A^{\mu;\mu;\mu}\right)_{;\mu} = -\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x^{\mu}} + \frac{\partial}{\partial x^{\sigma}}\left(\frac{\partial A^{\mu}}{\partial x^{\mu}}\frac{1}{2}\frac{\partial g^{\sigma\mu}}{\partial x_{\mu}}\right).... \quad (289)$$

Because two same contravariant index existed in $-\left(A^{\mu;\mu;\mu}\right)_{;\mu}$ of (286), (287), (288), (289), I decided not to handle (286), (287), (288), (289). Furthermore, I rewrite index ν of the coordinate systems of (110), (111), (112), (113) by Definision 3, Definision 2 and get

$$-\left(A^{\mu;\mu}\right)^{;\mu}_{;\mu} = -\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x_{\mu}} + \frac{\partial}{\partial x_{\mu}}\left(\frac{\partial A^{\mu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_{\mu}}\right)\right)..., \quad (290)$$

$$-\left(A^{\mu;\mu}\right)^{;\mu}_{;\mu} = -\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x_{\mu}} + \frac{\partial}{\partial x_{\mu}}\left(\frac{\partial A^{\mu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\sigma}}\right)\right)..., \quad (291)$$

$$-\left(A^{\mu;\mu}\right)^{;\mu}_{;\mu} = -\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x_{\mu}} + \frac{\partial}{\partial x_{\mu}}\left(\frac{\partial A^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\mu}}\right)\right)..., \quad (292)$$

$$-\left(A^{\mu;\mu}\right)^{;\mu}_{;\mu} = -\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x_{\mu}} + \frac{\partial}{\partial x^{\sigma}}\left(\frac{\partial A^{\mu}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\mu}}\right)\right).... \quad (293)$$

Because two same contravariant index existed in $-\left(A^{\mu;\mu}\right)^{;\mu}_{;\mu}$ of (290), (291), (292), (293), I decided not to handle (290), (291), (292), (293).

Similarly, I rewrite index ν of the coordinate systems of (114), (115), (116), (117) by Definision 3 and get

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}-\frac{\partial^2 A^{\sigma}}{\partial x^{\mu}\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)..., \quad (294)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}-\frac{\partial^2 A^{\sigma}}{\partial x^{\mu}\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)..., \quad (295)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}-\frac{\partial^2 A^{\sigma}}{\partial x^{\sigma}\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)..., \quad (296)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}-\frac{\partial^2 A^{\sigma}}{\partial x^{\mu}\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)..., \quad (297)$$

Because two same covariant index existed in $-A^{\mu}_{;\mu;\mu;\mu}$ of (294), (295), (296), (297), I decided not to handle (294), (295), (296), (297). Furthermore, I rewrite index ν of the coordinate systems of (114), (115), (116), (117) by Definision 2 and get

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}+\frac{\partial^2 A^{\sigma}}{\partial x_{\mu}\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)..., \quad (298)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}+\frac{\partial^2 A^{\sigma}}{\partial x_{\mu}\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)..., \quad (299)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}+\frac{\partial^2 A^{\sigma}}{\partial x^{\sigma}\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x_{\mu}}\right)..., \quad (300)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}+\frac{\partial^2 A^{\sigma}}{\partial x_{\mu}\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x_{\mu}}\right).... \quad (301)$$

Because two same contravariant index existed in $A^{\mu;\mu;\mu;\mu}$ of (298), (299), (300), (301), I decided not to handle (298), (299), (300), (301). Furthermore, I rewrite index ν of the coordinate systems of (114), (115), (116), (117) by Definision 3, Definision 2 and get

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial^2 A^{\sigma}}{\partial x^{\mu}\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)..., \quad (302)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial^2 A^{\sigma}}{\partial x^{\mu}\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)..., \quad (303)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial^2 A^{\sigma}}{\partial x^{\sigma}\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)..., \quad (304)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial^2 A^{\sigma}}{\partial x_{\mu}\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right).... \quad (305)$$

There isn't index ν in the second term of the right side of (302), (303), (304), (305) at all here. Therefore, I rewrite dummy index σ of (302), (303), (304), (305) in dummy index ν and can get

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial^2 A^{\nu}}{\partial x^{\mu}\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\right)..., \quad (306)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial^2 A^{\nu}}{\partial x^{\mu}\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}\right)...,\quad(307)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial^2 A^{\nu}}{\partial x^{\nu}\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)...,\quad(308)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial^2 A^{\nu}}{\partial x_{\mu}\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)....\quad(309)$$

Furthermore, I rewrite index ν of the coordinate systems of (114), (115), (116), (117) by Definision 3, Definision 2 and get

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial^2 A^{\sigma}}{\partial x_{\mu}\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)...,\quad(310)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial^2 A^{\sigma}}{\partial x_{\mu}\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)...,\quad(311)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial^2 A^{\sigma}}{\partial x^{\sigma}\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)...,\quad(312)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial^2 A^{\sigma}}{\partial x^{\mu}\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)....\quad(313)$$

There isn't index ν in the second term of the right side of (310), (311), (312), (313) at all here. Therefore, I rewrite dummy index σ of (310), (311), (312), (313) in dummy index ν and can get

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial^2 A^{\nu}}{\partial x_{\mu}\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\right)...,\quad(314)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial^2 A^{\nu}}{\partial x_{\mu}\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}\right)...,\quad(315)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial^2 A^{\nu}}{\partial x^{\nu}\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)...,\quad(316)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial^2 A^{\nu}}{\partial x^{\mu}\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)....\quad(317)$$

Furthermore, I rewrite index ν of the coordinate systems of (114), (115), (116), (117) by Definision 3, Definision 2 and get

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}}+\frac{\partial^2 A^{\sigma}}{\partial x^{\mu}\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)...,\quad(318)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}}+\frac{\partial^2 A^{\sigma}}{\partial x^{\mu}\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)...,\quad(319)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}}+\frac{\partial^2 A^{\sigma}}{\partial x^{\sigma}\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\mu}}\right)...,\quad(320)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_\mu\partial x^\mu\partial x^\mu}+\frac{\partial^2 A^{\sigma}}{\partial x^\mu\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\mu}\right)\text{....}\quad(321)$$

There isn't index ν in the second term of the right side of (318), (319), (320), (321) at all here. Therefore, I rewrite dummy index σ of (318), (319), (320), (321) in dummy index ν and can get

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_\mu\partial x^\mu\partial x^\mu}+\frac{\partial^2 A^{\nu}}{\partial x^\mu\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x_\mu}\right)\text{...,}\quad(322)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_\mu\partial x^\mu\partial x^\mu}+\frac{\partial^2 A^{\nu}}{\partial x^\mu\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\nu}\right)\text{...,}\quad(323)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_\mu\partial x^\mu\partial x^\mu}+\frac{\partial^2 A^{\nu}}{\partial x^\nu\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\mu}\right)\text{...,}\quad(324)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_\mu\partial x^\mu\partial x^\mu}+\frac{\partial^2 A^{\nu}}{\partial x^\mu\partial x^\nu}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\mu}\right)\text{....}\quad(325)$$

By the way, by establishment of $A^{\mu}_{;\dot\nu;\ddot\nu;\hat\nu}=A^{\mu}_{;\dot\nu;\hat\nu;\ddot\nu}=A^{\mu}_{;\hat\nu;\dot\nu;\ddot\nu}=A^{\mu}_{;\ddot\nu;\dot\nu;\hat\nu}=A^{\mu}_{;\ddot\nu;\hat\nu;\dot\nu}=A^{\mu}_{;\hat\nu;\ddot\nu;\dot\nu}$, (306), (314), (322) must be equal each other. (306), (314), (322) are equal each other in consideration of Definision 5. Furthermore, by establishment of $A^{\mu}_{;\dot\nu;\ddot\nu;\hat\nu}=A^{\mu}_{;\dot\nu;\hat\nu;\ddot\nu}=A^{\mu}_{;\hat\nu;\dot\nu;\ddot\nu}=A^{\mu}_{;\ddot\nu;\dot\nu;\hat\nu}=A^{\mu}_{;\ddot\nu;\hat\nu;\dot\nu}=A^{\mu}_{;\hat\nu;\ddot\nu;\dot\nu}$, (307), (315), (323) must be equal each other. (307), (315), (323) aren't equal each other in consideration of Definision5. Thus, I decide not to handle (307), (315), (323). Furthermore, by establishment of $A^{\mu}_{;\dot\nu;\ddot\nu;\hat\nu}=A^{\mu}_{;\dot\nu;\hat\nu;\ddot\nu}=A^{\mu}_{;\hat\nu;\dot\nu;\ddot\nu}=A^{\mu}_{;\ddot\nu;\dot\nu;\hat\nu}=A^{\mu}_{;\ddot\nu;\hat\nu;\dot\nu}=A^{\mu}_{;\hat\nu;\ddot\nu;\dot\nu}$, (308), (316), (324) must be equal each other. (308), (316), (324) aren't equal each other in consideration of Definision5. Thus, I decide not to handle (308), (316), (324). Furthermore, by establishment of $A^{\mu}_{;\dot\nu;\ddot\nu;\hat\nu}=A^{\mu}_{;\dot\nu;\hat\nu;\ddot\nu}=A^{\mu}_{;\hat\nu;\dot\nu;\ddot\nu}=A^{\mu}_{;\ddot\nu;\dot\nu;\hat\nu}=A^{\mu}_{;\ddot\nu;\hat\nu;\dot\nu}=A^{\mu}_{;\hat\nu;\ddot\nu;\dot\nu}$, (309), (317), (325) must be equal each other. (309), (317), (325)aren't equal each other in consideration of Definision 5. Thus, I decide not to handle (309), (317), (325). Furthermore, I rewrite index ν of the coordinate systems of (114), (115), (116), (117) by Definision 3, Definision 2 and get

$$-\left(A^{\mu}_{;\mu}\right)^{;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^\mu\partial x_\mu\partial x_\mu}-\frac{\partial^2 A^{\sigma}}{\partial x_\mu\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^\mu}\right)\text{...,}\quad(326)$$

$$-\left(A^{\mu}_{;\mu}\right)^{;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^\mu\partial x_\mu\partial x_\mu}-\frac{\partial^2 A^{\sigma}}{\partial x_\mu\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^\sigma}\right)\text{...,}\quad(327)$$

$$-\left(A^{\mu}_{;\mu}\right)^{;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^\mu\partial x_\mu\partial x_\mu}-\frac{\partial^2 A^{\sigma}}{\partial x^\sigma\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_\mu}\right)\text{...,}\quad(328)$$

$$-\left(A^{\mu}_{;\mu}\right)^{;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^\mu\partial x_\mu\partial x_\mu}-\frac{\partial^2 A^{\sigma}}{\partial x_\mu\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_\mu}\right)\text{....}\quad(329)$$

Because two same contravariant index existed in $-\left(A^{\mu}_{;\mu}\right)^{;\mu;\mu}$ of (326), (327), (328), (329), I decided not to handle (326), (327), (328), (329). Furthermore, I rewrite index ν of the coordinate systems of (114), (115), (116), (117) by Definision 3, Definision 2 and get

$$-\left(A^{\mu;\mu;\mu}\right)_{;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x_\mu\partial x_\mu\partial x^\mu}-\frac{\partial^2 A^{\sigma}}{\partial x_\mu\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_\mu}\right)\text{...,}\quad(330)$$

$$-\left(A^{\mu;\mu;\mu}\right)_{;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x_\mu\partial x_\mu\partial x^\mu}-\frac{\partial^2 A^{\sigma}}{\partial x_\mu\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\sigma}\right)\text{...,}\quad(331)$$

$$-\left(A^{\mu;\mu;\mu}\right)_{;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x^{\mu}}-\frac{\partial^2 A^{\sigma}}{\partial x^{\sigma}\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x_{\mu}}\right)...,\quad (332)$$

$$-\left(A^{\mu;\mu;\mu}\right)_{;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x^{\mu}}-\frac{\partial^2 A^{\sigma}}{\partial x^{\mu}\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x_{\mu}}\right)....\quad (333)$$

Because two same contravariant index existed in $-\left(A^{\mu;\mu;\mu}\right)_{;\mu}$ of (330), (331), (332), (333), I decided not to handle (330), (331), (332), (333). Furthermore, I rewrite index ν of the coordinate systems of (114), (115), (116), (117) by Definision 3, Definision 2 and get

$$-\left(A^{\mu;\mu}\right)_{;\mu}^{;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x_{\mu}}-\frac{\partial^2 A^{\sigma}}{\partial x^{\mu}\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g_{\sigma}^{\mu}}{\partial x_{\mu}}\right)...,\quad (334)$$

$$-\left(A^{\mu;\mu}\right)_{;\mu}^{;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x_{\mu}}-\frac{\partial^2 A^{\sigma}}{\partial x^{\mu}\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)...,\quad (335)$$

$$-\left(A^{\mu;\mu}\right)_{;\mu}^{;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x_{\mu}}-\frac{\partial^2 A^{\sigma}}{\partial x^{\sigma}\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\mu}}\right)...,\quad (336)$$

$$-\left(A^{\mu;\mu}\right)_{;\mu}^{;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x_{\mu}}-\frac{\partial^2 A^{\sigma}}{\partial x_{\mu}\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\mu}}\right)....\quad (337)$$

Because two same contravariant index existed in $-\left(A^{\mu;\mu}\right)_{;\mu}^{;\mu}$ of (334), (335), (336), (337), I decided not to handle (334), (335), (336), (337). Similarly, I rewrite index ν of the coordinate systems of (118) ~ (127) by Definision 3 and get

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}-\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g_{\mu}^{\sigma}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g_{\sigma}^{\mu}}{\partial x^{\sigma}}\right)...,\quad (338)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}-\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g_{\sigma}^{\sigma}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g_{\sigma}^{\mu}}{\partial x^{\mu}}\right)...,\quad (339)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}-\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g_{\mu}^{\sigma}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g_{\sigma}^{\mu}}{\partial x^{\mu}}\right)...,\quad (340)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}-\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g_{\mu}^{\sigma}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g_{\sigma}^{\mu}}{\partial x^{\mu}}\right)...,\quad (341)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}-\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g_{\sigma}^{\sigma}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g_{\mu}^{\mu}}{\partial x^{\sigma}}\right)...,\quad (342)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}-\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g_{\mu}^{\sigma}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g_{\mu}^{\mu}}{\partial x^{\sigma}}\right)...,\quad (343)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}-\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g_{\mu}^{\sigma}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g_{\mu}^{\mu}}{\partial x^{\sigma}}\right)...,\quad (344)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}-\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)..., \quad (345)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}-\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)..., \quad (346)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}-\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right).... \quad (347)$$

Because two same covariant index existed in $-A^{\mu}_{;\mu;\mu;\mu}$ of (338) ~ (347), I decided not to handle (338) ~ (347). Furthermore, I rewrite index ν of the coordinate systems of (118) ~ (127) by Definision 2 and get

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\right)..., \quad (348)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)..., \quad (349)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)..., \quad (350)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)..., \quad (351)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)..., \quad (352)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)..., \quad (353)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)..., \quad (354)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x_{\mu}}\right)..., \quad (355)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x_{\mu}}\right)..., \quad (356)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x_{\mu}}\right)..., \quad (357)$$

Because two same contravariant index existed in $A^{\mu;\mu;\mu;\mu}$ of (348) ~ (357), I decided not to handle (348) ~ (357). Furthermore, I rewrite index ν of the coordinate systems of (118) ~ (127) by Definision 3, Definision 2 and get

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\right)..., \quad (358)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)..., \quad (359)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)..., \quad (360)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)..., \quad (361)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)..., \quad (362)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)..., \quad (363)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)..., \quad (364)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)..., \quad (365)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)..., \quad (366)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right).... \quad (367)$$

There isn't index ν in the second term of the right side of (358) ~ (367) at all here. Therefore, I rewrite dummy index σ of (358) ~ (367) in dummy index ν and can get

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x_{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\right)..., \quad (368)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x_{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\right)..., \quad (369)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x_{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\right)..., \quad (370)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x^{\nu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\right)..., \quad (371)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x_{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}\right)..., \quad (372)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x_{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}\right)..., \quad (373)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x^{\nu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}\right)..., \quad (374)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x_{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\nu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)..., \quad (375)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x^{\nu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)..., \quad (376)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+\frac{\partial}{\partial x^{\nu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^{\nu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right).... \quad (377)$$

Furthermore, I rewrite index ν of the coordinate systems of (118) ~ (127) by Definision 3, Definision 2 and get

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\right)..., \quad (378)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)..., \quad (379)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)..., \quad (380)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)..., \quad (381)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)..., \quad (382)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)..., \quad (383)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)..., \quad (384)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)..., \quad (385)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)..., \quad (386)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right).... \quad (387)$$

There isn't index ν in the second term of the right side of (378) ~ (387) at all here. Therefore, I rewrite dummy index σ of (378) ~ (387) in dummy index ν and can get

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\right)..., \quad (388)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\right)\ldots,\quad(389)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^{\nu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\right)\ldots,\quad(390)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\nu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\right)\ldots,\quad(391)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}\right)\ldots,\quad(392)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^{\nu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}\right)\ldots,\quad(393)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\nu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}\right)\ldots,\quad(394)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\nu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)\ldots,\quad(395)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\nu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)\ldots,\quad(396)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\nu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)\ldots.\quad(397)$$

Furthermore, I rewrite index ν of the coordinate systems of (118) ~ (127) by Definision 3, Definision 2 and get

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\right)\ldots,\quad(398)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)\ldots,\quad(399)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)\ldots,\quad(400)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)\ldots,\quad(401)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)\ldots,\quad(402)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)\ldots,\quad(403)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}}+\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)\ldots,\quad(404)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu\partial x^\mu\partial x^\mu}+\frac{\partial}{\partial x^\mu}\left(A^\sigma\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\mu}\right)...,\quad(405)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu\partial x^\mu\partial x^\mu}+\frac{\partial}{\partial x^\sigma}\left(A^\sigma\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\mu}\right)...,\quad(406)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu\partial x^\mu\partial x^\mu}+\frac{\partial}{\partial x^\sigma}\left(A^\sigma\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\mu}\right)....\quad(407)$$

There isn't index ν in the second term of the right side of (398) ~ (407) at all here. Therefore, I rewrite dummy index σ of (398) ~ (407) in dummy index ν and can get

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu\partial x^\mu\partial x^\mu}+\frac{\partial}{\partial x^\mu}\left(A^\nu\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g_\nu^\mu}{\partial x^\nu}\right)...,\quad(408)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu\partial x^\mu\partial x^\mu}+\frac{\partial}{\partial x^\mu}\left(A^\nu\frac{1}{2}\left(\frac{\partial g_\nu^\nu}{\partial x^\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g_\nu^\mu}{\partial x_\mu}\right)...,\quad(409)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu\partial x^\mu\partial x^\mu}+\frac{\partial}{\partial x^\mu}\left(A^\nu\frac{1}{2}\left(\frac{\partial g_\mu^\nu}{\partial x^\nu}\right)\right)\frac{1}{2}\left(\frac{\partial g_\nu^\mu}{\partial x_\mu}\right)...,\quad(410)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu\partial x^\mu\partial x^\mu}+\frac{\partial}{\partial x^\nu}\left(A^\nu\frac{1}{2}\left(\frac{\partial g_\mu^\nu}{\partial x^\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g_\nu^\mu}{\partial x_\mu}\right)...,\quad(411)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu\partial x^\mu\partial x^\mu}+\frac{\partial}{\partial x^\mu}\left(A^\nu\frac{1}{2}\left(\frac{\partial g_\nu^\nu}{\partial x^\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\nu}\right)...,\quad(412)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu\partial x^\mu\partial x^\mu}+\frac{\partial}{\partial x^\mu}\left(A^\nu\frac{1}{2}\left(\frac{\partial g_\mu^\nu}{\partial x^\nu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\nu}\right)...,\quad(413)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu\partial x^\mu\partial x^\mu}+\frac{\partial}{\partial x^\nu}\left(A^\nu\frac{1}{2}\left(\frac{\partial g_\mu^\nu}{\partial x^\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\nu}\right)...,\quad(414)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu\partial x^\mu\partial x^\mu}+\frac{\partial}{\partial x^\mu}\left(A^\nu\frac{1}{2}\left(\frac{\partial g_\nu^\nu}{\partial x^\nu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\mu}\right)...,\quad(415)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu\partial x^\mu\partial x^\mu}+\frac{\partial}{\partial x^\nu}\left(A^\nu\frac{1}{2}\left(\frac{\partial g_\nu^\nu}{\partial x^\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\mu}\right)...,\quad(416)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu\partial x^\mu\partial x^\mu}+\frac{\partial}{\partial x^\nu}\left(A^\nu\frac{1}{2}\left(\frac{\partial g_\mu^\nu}{\partial x^\nu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\mu}\right)....\quad(417)$$

Furthermore, I rewrite index ν of the coordinate systems of (118) ~(127) by Definision 3, Definision 2 and get

$$-\left(A_{;\mu}^{\mu}\right)^{;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu\partial x_\mu\partial x_\mu}-\frac{\partial}{\partial x_\mu}\left(A^\sigma\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x_\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\mu}{\partial x^\sigma}\right)...,\quad(418)$$

$$-\left(A_{;\mu}^{\mu}\right)^{;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu\partial x_\mu\partial x_\mu}-\frac{\partial}{\partial x_\mu}\left(A^\sigma\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x_\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\mu}{\partial x^\mu}\right)...,\quad(419)$$

$$-\left(A^{\mu}_{;\mu}\right)^{;\mu;\mu} = -\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu} - \frac{\partial}{\partial x_\mu}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^\mu}\right)..., \quad (420)$$

$$-\left(A^{\mu}_{;\mu}\right)^{;\mu;\mu} = \frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu} - \frac{\partial}{\partial x^\sigma}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^\mu}\right)..., \quad (421)$$

$$-\left(A^{\mu}_{;\mu}\right)^{;\mu;\mu} = -\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu} - \frac{\partial}{\partial x_\mu}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^\sigma}\right)..., \quad (422)$$

$$-\left(A^{\mu}_{;\mu}\right)^{;\mu;\mu} = \frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu} - \frac{\partial}{\partial x_\mu}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^\sigma}\right)..., \quad (423)$$

$$-\left(A^{\mu}_{;\mu}\right)^{;\mu;\mu} = -\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu} - \frac{\partial}{\partial x^\sigma}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^\sigma}\right)..., \quad (424)$$

$$-\left(A^{\mu}_{;\mu}\right)^{;\mu;\mu} = -\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu} - \frac{\partial}{\partial x_\mu}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_\mu}\right)..., \quad (425)$$

$$-\left(A^{\mu}_{;\mu}\right)^{;\mu;\mu} = -\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu} - \frac{\partial}{\partial x^\sigma}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_\mu}\right)..., \quad (426)$$

$$-\left(A^{\mu}_{;\mu}\right)^{;\mu;\mu} = -\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu} - \frac{\partial}{\partial x^\sigma}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_\mu}\right).... \quad (427)$$

Because two same contravariant index existed in $-\left(A^{\mu}_{;\mu}\right)^{;\mu;\mu}$ of (418) ~ (427), I decided not to handle (418) ~ (427). Furthermore, I rewrite index ν of the coordinate systems of (118) ~ (127) by Definision 3, Definision 2 and get

$$-\left(A^{\mu;\mu;\mu}\right)_{;\mu} = -\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu} - \frac{\partial}{\partial x^\mu}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^\sigma}\right)..., \quad (428)$$

$$-\left(A^{\mu;\mu;\mu}\right)_{;\mu} = -\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu} - \frac{\partial}{\partial x^\mu}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_\mu}\right)..., \quad (429)$$

$$-\left(A^{\mu;\mu;\mu}\right)_{;\mu} = -\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu} - \frac{\partial}{\partial x^\mu}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_\mu}\right)..., \quad (430)$$

$$-\left(A^{\mu;\mu;\mu}\right)_{;\mu} = -\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu} - \frac{\partial}{\partial x^\sigma}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_\mu}\right)..., \quad (431)$$

$$-\left(A^{\mu;\mu;\mu}\right)_{;\mu} = -\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu} - \frac{\partial}{\partial x^\mu}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\sigma}\right)..., \quad (432)$$

$$-\left(A^{\mu;\mu;\mu}\right)_{;\mu} = -\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu} - \frac{\partial}{\partial x^\mu}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\sigma}\right)..., \quad (433)$$

$$-\left(A^{\mu;\mu;\mu}\right)_{;\mu} = -\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu} - \frac{\partial}{\partial x^\sigma}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\sigma}\right)..., \quad (434)$$

$$-\left(A^{\mu;\mu;\mu}\right)_{;\mu} = -\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu} - \frac{\partial}{\partial x^\mu}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x_\mu}\right)..., \quad (435)$$

$$-\left(A^{\mu;\mu;\mu}\right)_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}-\frac{\partial}{\partial x^\sigma}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x_\mu}\right)..., \quad (436)$$

$$-\left(A^{\mu;\mu;\mu}\right)_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}-\frac{\partial}{\partial x^\sigma}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x_\mu}\right).... \quad (437)$$

Because two same contravariant index existed in $-\left(A^{\mu;\mu;\mu}\right)_{;\mu}$ of (428) ~ (437), I decided not to handle (428) ~ (437). Furthermore, I rewrite index ν of the coordinate systems of (118) ~ (127) by Definision 3, Definision 2 and get

$$-\left(A^{\mu;\mu}\right)_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}-\frac{\partial}{\partial x_\mu}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\mu}{\partial x^\sigma}\right)..., \quad (438)$$

$$-\left(A^{\mu;\mu}\right)_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}-\frac{\partial}{\partial x_\mu}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\mu}{\partial x_\mu}\right)..., \quad (439)$$

$$-\left(A^{\mu;\mu}\right)_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}-\frac{\partial}{\partial x_\mu}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\mu}{\partial x_\mu}\right)..., \quad (440)$$

$$-\left(A^{\mu;\mu}\right)_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}-\frac{\partial}{\partial x^\sigma}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x_\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\mu}{\partial x_\mu}\right)..., \quad (441)$$

$$-\left(A^{\mu;\mu}\right)_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}-\frac{\partial}{\partial x_\mu}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\sigma}\right)..., \quad (442)$$

$$-\left(A^{\mu;\mu}\right)_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}-\frac{\partial}{\partial x_\mu}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\sigma}\right)..., \quad (443)$$

$$-\left(A^{\mu;\mu}\right)_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}-\frac{\partial}{\partial x^\sigma}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x_\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\sigma}\right)..., \quad (444)$$

$$-\left(A^{\mu;\mu}\right)_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}-\frac{\partial}{\partial x_\mu}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\mu}\right)..., \quad (445)$$

$$-\left(A^{\mu;\mu}\right)_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}-\frac{\partial}{\partial x^\sigma}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x_\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\mu}\right)..., \quad (446)$$

$$-\left(A^{\mu;\mu}\right)_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}-\frac{\partial}{\partial x^\sigma}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\mu}\right).... \quad (447)$$

Because two same contravariant index existed in $-\left(A^{\mu;\mu}\right)_{;\mu}^{;\mu}$ of (438) ~ (447), I decided not to handle (438)~ (447). Similarly, I rewrite index ν of the coordinate systems of (128)~ (137) by Definision 3 and get

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x^\mu}+2\frac{\partial A^\sigma}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\mu}{\partial x^\sigma}\right)..., \quad (448)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x^\mu}+2\frac{\partial A^\sigma}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\mu}{\partial x^\mu}\right)..., \quad (449)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}+2\frac{\partial A^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)..., \quad (450)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}+2\frac{\partial A^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)..., \quad (451)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}+2\frac{\partial A^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)..., \quad (452)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}+2\frac{\partial A^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)..., \quad (453)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}+2\frac{\partial A^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)..., \quad (454)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}+2\frac{\partial A^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)..., \quad (455)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}+2\frac{\partial A^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)..., \quad (456)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}+2\frac{\partial A^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right).... \quad (457)$$

Because two same covariant index existed in $-A^{\mu}_{;\mu;\mu;\mu}$ of (448) ~ (457), I decided not to handle (448)~ (457). Furthermore, I rewrite index ν of the coordinate systems of (128) ~ (137) by Definision 2 and get

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}-2\frac{\partial A^{\sigma}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\right)..., \quad (458)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}-2\frac{\partial A^{\sigma}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)..., \quad (459)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}-2\frac{\partial A^{\sigma}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)..., \quad (460)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}-2\frac{\partial A^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)..., \quad (461)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}-2\frac{\partial A^{\sigma}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)..., \quad (462)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}-2\frac{\partial A^{\sigma}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)..., \quad (463)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}-2\frac{\partial A^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)..., \quad (464)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}-2\frac{\partial A^{\sigma}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x_{\mu}}\right)..., \quad (465)$$

$$A^{\mu;\mu;\mu;\mu} = \frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x_\mu} - 2\frac{\partial A^\sigma}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x_\mu}\right)..., \quad (466)$$

$$A^{\mu;\mu;\mu;\mu} = \frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x_\mu} - 2\frac{\partial A^\sigma}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x_\mu}\right).... \quad (467)$$

Because two same contravariant index existed in $A^{\mu;\mu;\mu;\mu}$ of (458) ~ (467), I decided not to handle (458) ~ (467). Furthermore, I rewrite index ν of the coordinate systems of (128) ~ (137) by Definision 3, Definision 2 and get

$$\left(A^\mu_{;\mu;\mu}\right)^{;\mu} = \frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x_\mu} - 2\frac{\partial A^\sigma}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\mu}{\partial x^\sigma}\right)..., \quad (468)$$

$$\left(A^\mu_{;\mu;\mu}\right)^{;\mu} = \frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x_\mu} - 2\frac{\partial A^\sigma}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\mu}{\partial x^\mu}\right)..., \quad (469)$$

$$\left(A^\mu_{;\mu;\mu}\right)^{;\mu} = \frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x_\mu} - 2\frac{\partial A^\sigma}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\mu}{\partial x^\mu}\right)..., \quad (470)$$

$$\left(A^\mu_{;\mu;\mu}\right)^{;\mu} = \frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x_\mu} - 2\frac{\partial A^\sigma}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\mu}{\partial x^\mu}\right)..., \quad (471)$$

$$\left(A^\mu_{;\mu;\mu}\right)^{;\mu} = \frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x_\mu} - 2\frac{\partial A^\sigma}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g_\mu^\mu}{\partial x^\sigma}\right)..., \quad (472)$$

$$\left(A^\mu_{;\mu;\mu}\right)^{;\mu} = \frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x_\mu} - 2\frac{\partial A^\sigma}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g_\mu^\mu}{\partial x^\sigma}\right)..., \quad (473)$$

$$\left(A^\mu_{;\mu;\mu}\right)^{;\mu} = \frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x_\mu} - 2\frac{\partial A^\sigma}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g_\mu^\mu}{\partial x^\sigma}\right)..., \quad (474)$$

$$\left(A^\mu_{;\mu;\mu}\right)^{;\mu} = \frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x_\mu} - 2\frac{\partial A^\sigma}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g_\mu^\mu}{\partial x^\mu}\right)..., \quad (475)$$

$$\left(A^\mu_{;\mu;\mu}\right)^{;\mu} = \frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x_\mu} - 2\frac{\partial A^\sigma}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g_\mu^\mu}{\partial x^\mu}\right)..., \quad (476)$$

$$\left(A^\mu_{;\mu;\mu}\right)^{;\mu} = \frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x_\mu} - 2\frac{\partial A^\sigma}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g_\mu^\mu}{\partial x^\mu}\right).... \quad (477)$$

There isn't index ν in the second term of the right side of (468) ~ (477) at all here. Therefore, I rewrite dummy index σ of (468) ~ (477) in dummy index ν and can get

$$\left(A^\mu_{;\mu;\mu}\right)^{;\mu} = \frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x_\mu} - 2\frac{\partial A^\nu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g_\mu^\nu}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g_\nu^\mu}{\partial x^\nu}\right)..., \quad (478)$$

$$\left(A^\mu_{;\mu;\mu}\right)^{;\mu} = \frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x_\mu} - 2\frac{\partial A^\nu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g_\nu^\nu}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g_\nu^\mu}{\partial x^\mu}\right)..., \quad (479)$$

$$\left(A^\mu_{;\mu;\mu}\right)^{;\mu} = \frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x_\mu} - 2\frac{\partial A^\nu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g_\mu^\nu}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g_\nu^\mu}{\partial x^\mu}\right)..., \quad (480)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}-2\frac{\partial A^{\nu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x_{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\right)..., \quad (481)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}-2\frac{\partial A^{\nu}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}\right)..., \quad (482)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}-2\frac{\partial A^{\nu}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}\right)..., \quad (483)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}-2\frac{\partial A^{\nu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x_{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}\right)..., \quad (484)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}-2\frac{\partial A^{\nu}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)..., \quad (485)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}-2\frac{\partial A^{\nu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x_{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)..., \quad (486)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}-2\frac{\partial A^{\nu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right).... \quad (487)$$

Furthermore, I rewrite index ν of the coordinate systems of (128) ~ (137) by Definision 3, Definision 2 and get

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial A^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x_{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\right)..., \quad (488)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial A^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)..., \quad (489)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial A^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)..., \quad (490)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial A^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)..., \quad (491)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial A^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)..., \quad (492)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial A^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)..., \quad (493)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial A^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)..., \quad (494)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial A^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)..., \quad (495)$$

$$\left(A^{\mu}_{;\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial A^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)..., \quad (496)$$

$$\left(A^{\mu}_{,\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial A^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)\ldots,\quad (497)$$

There isn't index ν in the second term of the right side of (488) ~ (497) at all here. Therefore, I rewrite dummy index σ of (488) ~ (497) in dummy index ν and can get

$$\left(A^{\mu}_{,\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial A^{\nu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x_{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\right)\ldots,\quad (498)$$

$$\left(A^{\mu}_{,\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial A^{\nu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x_{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\right)\ldots,\quad (499)$$

$$\left(A^{\mu}_{,\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial A^{\nu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\right)\ldots,\quad (500)$$

$$\left(A^{\mu}_{,\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial A^{\nu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\right)\ldots,\quad (501)$$

$$\left(A^{\mu}_{,\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial A^{\nu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x_{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}\right)\ldots,\quad (502)$$

$$\left(A^{\mu}_{,\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial A^{\nu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}\right)\ldots,\quad (503)$$

$$\left(A^{\mu}_{,\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial A^{\nu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}\right)\ldots,\quad (504)$$

$$\left(A^{\mu}_{,\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial A^{\nu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)\ldots,\quad (505)$$

$$\left(A^{\mu}_{,\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial A^{\nu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)\ldots,\quad (506)$$

$$\left(A^{\mu}_{,\mu}\right)^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial A^{\nu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)\ldots.\quad (507)$$

Furthermore, I rewrite index ν of the coordinate systems of (128) ~ (137) by Definision 3, Definision 2 and get

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}}-2\frac{\partial A^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\right)\ldots,\quad (508)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}}-2\frac{\partial A^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)\ldots,\quad (509)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}}-2\frac{\partial A^{\sigma}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)\ldots,\quad (510)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}}-2\frac{\partial A^{\sigma}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)\ldots,\quad (511)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}-2\frac{\partial A^\sigma}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\sigma}\right)..., \quad (512)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}-2\frac{\partial A^\sigma}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\sigma}\right)..., \quad (513)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}-2\frac{\partial A^\sigma}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\sigma}\right)..., \quad (514)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}-2\frac{\partial A^\sigma}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\mu}\right)..., \quad (515)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}-2\frac{\partial A^\sigma}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\mu}\right)..., \quad (516)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}-2\frac{\partial A^\sigma}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\mu}\right).... \quad (517)$$

There isn't index ν in the second term of the right side of (508) ~ (517) at all here. Therefore, I rewrite dummy index σ of (508) ~ (517) in dummy index ν and can get

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}-2\frac{\partial A^\nu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g_\nu^\mu}{\partial x^\nu}\right)..., \quad (518)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}-2\frac{\partial A^\nu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g_\nu^\nu}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g_\nu^\mu}{\partial x_\mu}\right)..., \quad (519)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}-2\frac{\partial A^\nu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g_\mu^\nu}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g_\nu^\mu}{\partial x_\mu}\right)..., \quad (520)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}-2\frac{\partial A^\nu}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g_\mu^\nu}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g_\nu^\mu}{\partial x_\mu}\right)..., \quad (521)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}-2\frac{\partial A^\nu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g_\nu^\nu}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\nu}\right)..., \quad (522)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}-2\frac{\partial A^\nu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g_\mu^\nu}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\nu}\right)..., \quad (523)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}-2\frac{\partial A^\nu}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g_\mu^\nu}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\nu}\right)..., \quad (524)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}-2\frac{\partial A^\nu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g_\nu^\nu}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\mu}\right)..., \quad (525)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}-2\frac{\partial A^\nu}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g_\nu^\nu}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\mu}\right)..., \quad (526)$$

$$\left(A^{\mu;\mu}\right)_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}-2\frac{\partial A^\nu}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g_\mu^\nu}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\mu}\right).... \quad (527)$$

Furthermore, I rewrite index ν of the coordinate systems of (128) ~ (137) by Definision 3, Definision 2 and get

$$-\left(A^\mu_{;\mu}\right)^{;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu}+2\frac{\partial A^\sigma}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\mu}{\partial x^\sigma}\right)..., \quad (528)$$

$$-\left(A^\mu_{;\mu}\right)^{;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu}+2\frac{\partial A^\sigma}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\mu}{\partial x^\mu}\right)..., \quad (529)$$

$$-\left(A^\mu_{;\mu}\right)^{;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu}+2\frac{\partial A^\sigma}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\mu}{\partial x^\mu}\right)..., \quad (530)$$

$$-\left(A^\mu_{;\mu}\right)^{;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu}+2\frac{\partial A^\sigma}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\mu}{\partial x^\mu}\right)..., \quad (531)$$

$$-\left(A^\mu_{;\mu}\right)^{;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu}+2\frac{\partial A^\sigma}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g_\mu^\mu}{\partial x^\sigma}\right)..., \quad (532)$$

$$-\left(A^\mu_{;\mu}\right)^{;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu}+2\frac{\partial A^\sigma}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g_\mu^\mu}{\partial x^\sigma}\right)..., \quad (533)$$

$$-\left(A^\mu_{;\mu}\right)^{;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu}+2\frac{\partial A^\sigma}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g_\mu^\mu}{\partial x^\sigma}\right)..., \quad (534)$$

$$-\left(A^\mu_{;\mu}\right)^{;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu}+2\frac{\partial A^\sigma}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g_\mu^\mu}{\partial x_\mu}\right)..., \quad (535)$$

$$-\left(A^\mu_{;\mu}\right)^{;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu}+2\frac{\partial A^\sigma}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g_\mu^\mu}{\partial x_\mu}\right)..., \quad (536)$$

$$-\left(A^\mu_{;\mu}\right)^{;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu}+2\frac{\partial A^\sigma}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g_\mu^\mu}{\partial x_\mu}\right).... \quad (537)$$

Because two same contravariant index existed in $-\left(A^\mu_{;\mu}\right)^{;\mu;\mu}$ of (528) ~ (537), I decided not to handle (528) ~ (537). Furthermore, I rewrite index ν of the coordinate systems of (128) ~ (137) by Definision 3, Definision 2 and get

$$-\left(A^{\mu;\mu;\mu}\right)_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}+2\frac{\partial A^\sigma}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\mu}{\partial x^\sigma}\right)..., \quad (538)$$

$$-\left(A^{\mu;\mu;\mu}\right)_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}+2\frac{\partial A^\sigma}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\mu}{\partial x_\mu}\right)..., \quad (539)$$

$$-\left(A^{\mu;\mu;\mu}\right)_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}+2\frac{\partial A^\sigma}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\mu}{\partial x_\mu}\right)..., \quad (540)$$

$$-\left(A^{\mu;\mu;\mu}\right)_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}+2\frac{\partial A^\sigma}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\mu}{\partial x_\mu}\right)..., \quad (541)$$

$$-\left(A^{\mu;\mu;\mu}\right)_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}+2\frac{\partial A^\sigma}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\sigma}\right)..., \quad (542)$$

$$-\left(A^{\mu;\mu;\mu}\right)_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}+2\frac{\partial A^\sigma}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\sigma}\right)..., \quad (543)$$

$$-\left(A^{\mu;\mu;\mu}\right)_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}+2\frac{\partial A^\sigma}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\sigma}\right)..., \quad (544)$$

$$-\left(A^{\mu;\mu;\mu}\right)_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}+2\frac{\partial A^\sigma}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x_\mu}\right)..., \quad (545)$$

$$-\left(A^{\mu;\mu;\mu}\right)_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}+2\frac{\partial A^\sigma}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x_\mu}\right)..., \quad (546)$$

$$-\left(A^{\mu;\mu;\mu}\right)_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}+2\frac{\partial A^\sigma}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x_\mu}\right).... \quad (547)$$

Because two same contravariant index existed in $-\left(A^{\mu;\mu;\mu}\right)_{;\mu}$ of (538) ~ (547), I decided not to handle (538) ~ (547). Furthermore, I rewrite index ν of the coordinate systems of (128) ~ (137) by Definision 3, Definision 2 and get

$$-\left(A^{\mu;\mu}\right)_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}+2\frac{\partial A^\sigma}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\mu}{\partial x^\sigma}\right)..., \quad (548)$$

$$-\left(A^{\mu;\mu}\right)_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}+2\frac{\partial A^\sigma}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\mu}{\partial x_\mu}\right)..., \quad (549)$$

$$-\left(A^{\mu;\mu}\right)_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}+2\frac{\partial A^\sigma}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\mu}{\partial x_\mu}\right)..., \quad (550)$$

$$-\left(A^{\mu;\mu}\right)_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}+2\frac{\partial A^\sigma}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\mu}{\partial x_\mu}\right)..., \quad (551)$$

$$-\left(A^{\mu;\mu}\right)_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}+2\frac{\partial A^\sigma}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\sigma}\right)..., \quad (552)$$

$$-\left(A^{\mu;\mu}\right)_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}+2\frac{\partial A^\sigma}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\sigma}\right)..., \quad (553)$$

$$-\left(A^{\mu;\mu}\right)_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}+2\frac{\partial A^\sigma}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\sigma}\right)..., \quad (554)$$

$$-\left(A^{\mu;\mu}\right)_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}+2\frac{\partial A^\sigma}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\mu}\right)..., \quad (555)$$

$$-\left(A^{\mu;\mu}\right)_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}+2\frac{\partial A^\sigma}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\mu}\right)..., \quad (556)$$

$$-\left(A^{\mu;\mu}\right)_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}+2\frac{\partial A^\sigma}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\mu}\right).... \quad (557)$$

Because two same contravariant index existed in $-\left(A^{\mu;\mu}\right)^{;\mu}_{;\mu}$ of (548) ~ (557), I decided not to handle (548) ~ (557). Similarly, I rewrite index ν of the coordinate systems of (138), (139), (140), (141) by Definision 3 and get

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}+2\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)..., \quad (558)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}+2\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)..., \quad (559)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}+2\frac{\partial^2 A^{\mu}}{\partial x^{\sigma}\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}\right)..., \quad (560)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}+2\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}\right).... \quad (561)$$

Because two same covariant index existed in $-A^{\mu}_{;\mu;\mu;\mu}$ of (558), (559), (560), (561), I decided not to handle (558), (559), (560), (561). Furthermore, I rewrite index ν of the coordinate systems of (138), (139), (140), (141) by Definision 2 and get

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}-2\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_{\mu}}\right)..., \quad (562)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}-2\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\sigma}}\right)..., \quad (563)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}-2\frac{\partial^2 A^{\mu}}{\partial x^{\sigma}\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_{\mu}}\right)..., \quad (564)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}-2\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_{\mu}}\right).... \quad (565)$$

Because two same contravariant index existed in $A^{\mu;\mu;\mu;\mu}$ of (562), (563), (564), (565), I decided not to handle (562), (563), (564), (565). Furthermore, I rewrite index ν of the coordinate systems of (138), (139), (140), (141) by Definision 3, Definision 2 and get

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}-2\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)..., \quad (566)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}-2\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)..., \quad (567)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}-2\frac{\partial^2 A^{\mu}}{\partial x^{\sigma}\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}\right)..., \quad (568)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}-2\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}\right).... \quad (569)$$

There isn't index ν in the second term of the right side of (566), (567), (568), (569) at all here. Therefore, I rewrite dummy index σ of (566), (567), (568), (569) in dummy index ν and can get

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu} = \frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}} - 2\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\right)..., \quad (570)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu} = \frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}} - 2\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\right)..., \quad (571)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu} = \frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}} - 2\frac{\partial^2 A^{\mu}}{\partial x^{\nu}\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu}}\right)..., \quad (572)$$

$$\left(A^{\mu}_{;\mu;\mu}\right)^{;\mu} = \frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}} - 2\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu}}\right).... \quad (573)$$

Furthermore, I rewrite index ν of the coordinate systems of (138), (139), (140), (141) by Definision 3, Definisions 2 and get

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu} = \frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}} - 2\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)..., \quad (574)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu} = \frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}} - 2\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)..., \quad (575)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu} = \frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}} - 2\frac{\partial^2 A^{\mu}}{\partial x^{\sigma}\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x_{\mu}}\right)..., \quad (576)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu} = \frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}} - 2\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x_{\mu}}\right).... \quad (577)$$

There isn't index ν in the second term of the right side of (574), (575), (576), (577) at all here. Therefore, I rewrite dummy index σ of (574), (575), (576), (577) in dummy index ν and can get

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu} = \frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}} - 2\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\right)..., \quad (578)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu} = \frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}} - 2\frac{\partial^2 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\right)..., \quad (579)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu} = \frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}} - 2\frac{\partial^2 A^{\mu}}{\partial x^{\nu}\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x_{\mu}}\right)..., \quad (580)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu} = \frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}} - 2\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x_{\mu}}\right).... \quad (581)$$

Furthermore, I rewrite index ν of the coordinate systems of (138), (139), (140), (141) by Definision 3, Definision 2 and get

$$(A^{\mu;\mu;})_{;\mu;\mu} = \frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}} - 2\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_{\mu}}\right)..., \quad (582)$$

$$(A^{\mu;\mu;})_{;\mu;\mu} = \frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}} - 2\frac{\partial^2 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\sigma}}\right)..., \quad (583)$$

$$(A^{\mu;\mu;})_{;\mu;\mu} = \frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu} - 2\frac{\partial^2 A^\mu}{\partial x^\sigma \partial x^\mu}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\mu}\right)..., \quad (584)$$

$$(A^{\mu;\mu;})_{;\mu;\mu} = \frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu} - 2\frac{\partial^2 A^\mu}{\partial x^\mu \partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\mu}\right).... \quad (585)$$

There isn't index ν in the second term of the right side of (582), (583), (584), (585) at all here. Therefore, I rewrite dummy index σ of (582), (583), (584), (585) in dummy index ν and can get

$$(A^{\mu;\mu;})_{;\mu;\mu} = \frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu} - 2\frac{\partial^2 A^\mu}{\partial x^\mu \partial x^\mu}\frac{1}{2}\left(\frac{\partial g^\nu_\nu}{\partial x_\mu}\right)..., \quad (586)$$

$$(A^{\mu;\mu;})_{;\mu;\mu} = \frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu} - 2\frac{\partial^2 A^\mu}{\partial x^\mu \partial x^\mu}\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^\nu}\right)..., \quad (587)$$

$$(A^{\mu;\mu;})_{;\mu;\mu} = \frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu} - 2\frac{\partial^2 A^\mu}{\partial x^\nu \partial x^\mu}\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^\mu}\right)..., \quad (588)$$

$$(A^{\mu;\mu;})_{;\mu;\mu} = \frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu} - 2\frac{\partial^2 A^\mu}{\partial x^\mu \partial x^\nu}\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^\mu}\right).... \quad (589)$$

By the way, by establishment of $A^\mu_{;\dot{\nu};\ddot{\nu};\hat{\nu}} = A^\mu_{;\dot{\nu};\hat{\nu};\ddot{\nu}} = A^\mu_{;\hat{\nu};\dot{\nu};\ddot{\nu}} = A^\mu_{;\ddot{\nu};\dot{\nu};\hat{\nu}} = A^\mu_{;\ddot{\nu};\hat{\nu};\dot{\nu}} = A^\mu_{;\hat{\nu};\ddot{\nu};\dot{\nu}}$, (570), (578), (586) must be equal each other. (570), (578), (586) are equal each other in consideration of Definision 5. Furthermore, by establishment of $A^\mu_{;\dot{\nu};\ddot{\nu};\hat{\nu}} = A^\mu_{;\dot{\nu};\hat{\nu};\ddot{\nu}} = A^\mu_{;\hat{\nu};\dot{\nu};\ddot{\nu}} = A^\mu_{;\ddot{\nu};\dot{\nu};\hat{\nu}} = A^\mu_{;\ddot{\nu};\hat{\nu};\dot{\nu}} = A^\mu_{;\hat{\nu};\ddot{\nu};\dot{\nu}}$, (571), (579), (587) must be equal each other. (571), (579), (587) aren't equal each other in consideration of Definision 5. Thus, I decide not to handle (571), (579), (587). Furthermore, by establishment of $A^\mu_{;\dot{\nu};\ddot{\nu};\hat{\nu}} = A^\mu_{;\dot{\nu};\hat{\nu};\ddot{\nu}} = A^\mu_{;\hat{\nu};\dot{\nu};\ddot{\nu}} = A^\mu_{;\ddot{\nu};\dot{\nu};\hat{\nu}} = A^\mu_{;\ddot{\nu};\hat{\nu};\dot{\nu}} = A^\mu_{;\hat{\nu};\ddot{\nu};\dot{\nu}}$, (572), (580), (588) must be equal each other. (572), (580), (588) aren't equal each other in consideration of Definision 5. Thus, I decide not to handle (572), (580), (588). Furthermore, by establishment of $A^\mu_{;\dot{\nu};\ddot{\nu};\hat{\nu}} = A^\mu_{;\dot{\nu};\hat{\nu};\ddot{\nu}} = A^\mu_{;\hat{\nu};\dot{\nu};\ddot{\nu}} = A^\mu_{;\ddot{\nu};\dot{\nu};\hat{\nu}} = A^\mu_{;\ddot{\nu};\hat{\nu};\dot{\nu}} = A^\mu_{;\hat{\nu};\ddot{\nu};\dot{\nu}}$, (573), (581), (589) must be equal each other. (573), (581), (589) aren't equal each other in consideration of Definision 5. Thus, I decide not to handle (573), (581), (589). Furthermore, I rewrite index ν of the coordinate systems of (138), (139), (140), (141) by Definision 3, Definision 2 and get

$$-(A^\mu_{;\mu})^{;\mu;\mu} = -\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu} + 2\frac{\partial^2 A^\mu}{\partial x_\mu \partial x_\mu}\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\mu}\right)..., \quad (590)$$

$$-(A^\mu_{;\mu})^{;\mu;\mu} = -\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu} + 2\frac{\partial^2 A^\mu}{\partial x_\mu \partial x_\mu}\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^\sigma}\right)..., \quad (591)$$

$$-(A^\mu_{;\mu})^{;\mu;\mu} = -\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu} + 2\frac{\partial^2 A^\mu}{\partial x^\sigma \partial x_\mu}\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x_\mu}\right)..., \quad (592)$$

$$-(A^\mu_{;\mu})^{;\mu;\mu} = -\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu} + 2\frac{\partial^2 A^\mu}{\partial x_\mu \partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x_\mu}\right).... \quad (593)$$

Because two same contravariant index existed in $-(A^\mu_{;\mu})^{;\mu;\mu}$ of (590), (591), (592), (593), I decided not to handle (590), (591), (592), (593). Furthermore, I rewrite index ν of the coordinate systems of (138), (139), (140), (141) by Definision 3, Definision 2 and get

$$-(A^{\mu;\mu;\mu})_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}+2\frac{\partial^2 A^\mu}{\partial x_\mu \partial x^\mu}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x_\mu}\right)\ldots, \quad (594)$$

$$-(A^{\mu;\mu;\mu})_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}+2\frac{\partial^2 A^\mu}{\partial x_\mu \partial x^\mu}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial g^\sigma}\right)\ldots, \quad (595)$$

$$-(A^{\mu;\mu;\mu})_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}+2\frac{\partial^2 A^\mu}{\partial x^\sigma \partial x^\mu}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_\mu}\right)\ldots, \quad (596)$$

$$-(A^{\mu;\mu;\mu})_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}+2\frac{\partial^2 A^\mu}{\partial x^\mu \partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_\mu}\right)\ldots. \quad (597)$$

Because two same contravariant index existed in - $(A^{\mu;\mu;\mu})_{;\mu}$ of (594), (595), (596), (597), I decided not to handle (594), (595), (596), (597). Furthermore, I rewrite index ν of the coordinate systems of (138), (139), (140), (141) by Definision 3, Definision 2 and get

$$-(A^{\mu;\mu})_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}+2\frac{\partial^2 A^\mu}{\partial x^\mu \partial x_\mu}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x_\mu}\right)\ldots, \quad (598)$$

$$-(A^{\mu;\mu})_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}+2\frac{\partial^2 A^\mu}{\partial x^\mu \partial x_\mu}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)\ldots, \quad (599)$$

$$-(A^{\mu;\mu})_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}+2\frac{\partial^2 A^\mu}{\partial x^\sigma \partial x_\mu}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\mu}\right)\ldots, \quad (600)$$

$$-(A^{\mu;\mu})_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}+2\frac{\partial^2 A^\mu}{\partial x_\mu \partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\mu}\right)\ldots. \quad (601)$$

Because two same contravariant index existed in - $(A^{\mu;\mu})_{;\mu}^{;\mu}$ of (598), (599), (600), (601), I decided not to handle (598), (599), (600), (601). Similarly, I rewrite index ν of the coordinate systems of (142) ~ (151) by Definision 3 and get

$$-A^\mu_{;\mu;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x^\mu}+2\frac{\partial}{\partial x^\mu}\left(A^\sigma\frac{1}{2}\left(\frac{\partial g_\mu^\mu}{\partial x^\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\sigma}\right)\ldots, \quad (602)$$

$$-A^\mu_{;\mu;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x^\mu}+2\frac{\partial}{\partial x^\mu}\left(A^\sigma\frac{1}{2}\left(\frac{\partial g_\sigma^\mu}{\partial x^\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\right)\ldots, \quad (603)$$

$$-A^\mu_{;\mu;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x^\mu}+2\frac{\partial}{\partial x^\mu}\left(A^\sigma\frac{1}{2}\left(\frac{\partial g_\mu^\mu}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\right)\ldots, \quad (604)$$

$$-A^\mu_{;\mu;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x^\mu}+2\frac{\partial}{\partial x^\sigma}\left(A^\sigma\frac{1}{2}\left(\frac{\partial g_\mu^\mu}{\partial x^\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\right)\ldots, \quad (605)$$

$$-A^\mu_{;\mu;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x^\mu}+2\frac{\partial}{\partial x^\mu}\left(A^\sigma\frac{1}{2}\left(\frac{\partial g_\sigma^\mu}{\partial x^\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\sigma}\right)\ldots, \quad (606)$$

$$-A^\mu_{;\mu;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x^\mu}+2\frac{\partial}{\partial x^\mu}\left(A^\sigma\frac{1}{2}\left(\frac{\partial g_\mu^\mu}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\sigma}\right)\ldots, \quad (607)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}+2\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)..., \quad (608)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}+2\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}\right)..., \quad (609)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}+2\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}\right)..., \quad (610)$$

$$-A^{\mu}_{;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x^{\mu}}+2\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}\right).... \quad (611)$$

Because two same covariant index existed in $-A^{\mu}_{;\mu;\mu;\mu}$ of (602) ~ (611), I decided not to handle (602) ~ (611). Furthermore, I rewrite index ν of the coordinate systems of (142) ~ (151) by Definision 2 and get

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}-2\frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\sigma}}\right)..., \quad (612)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}-2\frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_{\mu}}\right)..., \quad (613)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}-2\frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_{\mu}}\right)..., \quad (614)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}-2\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_{\mu}}\right)..., \quad (615)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}-2\frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\sigma}}\right)..., \quad (616)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}-2\frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\sigma}}\right)..., \quad (617)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}-2\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\sigma}}\right)..., \quad (618)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}-2\frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_{\mu}}\right)..., \quad (619)$$

$$A^{\mu;\mu;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}-2\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_{\mu}}\right)..., \quad (620)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x_{\mu}}-2\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_{\mu}}\right).... \quad (621)$$

Because two same contravariant index existed in $A^{\mu;\mu;\mu;\mu}$ of (612) ~ (621), I decided not to handle (612) ~ (621). Furthermore, I rewrite index ν of the coordinate systems of (142) ~ (151) by Definision 3, Definision 2 and get

$$(A^{\mu}_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}-2\frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\sigma}}\right)...,\quad(622)$$

$$(A^{\mu}_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}-2\frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)...,\quad(623)$$

$$(A^{\mu}_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}-2\frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)...,\quad(624)$$

$$(A^{\mu}_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}-2\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)...,\quad(625)$$

$$(A^{\mu}_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}-2\frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)...,\quad(626)$$

$$(A^{\mu}_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}-2\frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)...,\quad(627)$$

$$(A^{\mu}_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}-2\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)...,\quad(628)$$

$$(A^{\mu}_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}-2\frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}\right)...,\quad(629)$$

$$(A^{\mu}_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}-2\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}\right)...,\quad(630)$$

$$(A^{\mu}_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}-2\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\mu}}\right)....\quad(631)$$

There isn't index ν in the second term of the right side of (622) ~ (631) at all here. Therefore, I rewrite dummy index σ of (622) ~ (631) in dummy index ν and can get

$$(A^{\mu}_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}-2\frac{\partial}{\partial x_{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\nu}}\right)...,\quad(632)$$

$$(A^{\mu}_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}-2\frac{\partial}{\partial x_{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\right)...,\quad(633)$$

$$(A^{\mu}_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}-2\frac{\partial}{\partial x_{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\right)...,\quad(634)$$

$$(A^{\mu}_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}-2\frac{\partial}{\partial x^{\nu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\right)...,\quad(635)$$

$$(A^{\mu}_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}-2\frac{\partial}{\partial x_{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\right)...,\quad(636)$$

$$(A^{\mu}_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}-2\frac{\partial}{\partial x_{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\right)\ldots, \quad (637)$$

$$(A^{\mu}_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}-2\frac{\partial}{\partial x^{\nu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\right)\ldots, \quad (638)$$

$$(A^{\mu}_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}-2\frac{\partial}{\partial x_{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu}}\right)\ldots, \quad (639)$$

$$(A^{\mu}_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}-2\frac{\partial}{\partial x^{\nu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu}}\right)\ldots, \quad (640)$$

$$(A^{\mu}_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}-2\frac{\partial}{\partial x^{\nu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\nu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu}}\right)\ldots. \quad (641)$$

Furthermore, I rewrite index ν of the coordinate systems of (142) ~ (151) by Definision 3, Definision 2 and get

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\sigma}}\right)\ldots, \quad (642)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)\ldots, \quad (643)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)\ldots, \quad (644)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)\ldots, \quad (645)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)\ldots, \quad (646)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)\ldots, \quad (647)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)\ldots, \quad (648)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x_{\mu}}\right)\ldots, \quad (649)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x_{\mu}}\right)\ldots, \quad (650)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x_{\mu}}\right)\ldots. \quad (651)$$

Citation: Koji Ichidayama (2018) Tensor Satisfying Binary Law about the First, Second, Third-Order Covariant Derivative of the Contravariant Vector. SF J Nuclear Science 2:3.

There isn't index ν in the second term of the right side of (642) ~ (651) at all here. Therefore, I rewrite dummy index σ of (642) ~ (651) in dummy index ν and can get

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial}{\partial x^{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\nu}}\right)\ldots,\quad (652)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial}{\partial x^{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\right)\ldots,\quad (653)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial}{\partial x^{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\nu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\right)\ldots,\quad (654)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial}{\partial x^{\nu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\right)\ldots,\quad (655)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial}{\partial x^{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\right)\ldots,\quad (656)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial}{\partial x^{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\nu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\right)\ldots,\quad (657)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial}{\partial x^{\nu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\right)\ldots,\quad (658)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial}{\partial x^{\mu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\nu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x_{\mu}}\right)\ldots,\quad (659)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial}{\partial x^{\nu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\nu}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x_{\mu}}\right)\ldots,\quad (660)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}-2\frac{\partial}{\partial x^{\nu}}\left(A^{\nu}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\nu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x_{\mu}}\right)\ldots.\quad (661)$$

Furthermore, I rewrite index ν of the coordinate systems of (142) ~ (151) by Definision 3, Definision 2 and get

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}}-2\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\sigma}}\right)\ldots,\quad (662)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}}-2\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_{\mu}}\right)\ldots,\quad (663)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}}-2\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_{\mu}}\right)\ldots,\quad (664)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}}-2\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_{\mu}}\right)\ldots,\quad (665)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x^{\mu}\partial x^{\mu}}-2\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\sigma}}\right)\ldots,\quad (666)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}-2\frac{\partial}{\partial x^\mu}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^\mu_\mu}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)...,\quad (667)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}-2\frac{\partial}{\partial x^\sigma}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^\mu_\mu}{\partial x^\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)...,\quad (668)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}-2\frac{\partial}{\partial x^\mu}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\mu}\right)...,\quad (669)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}-2\frac{\partial}{\partial x^\sigma}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x^\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\mu}\right)...,\quad (670)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}-2\frac{\partial}{\partial x^\sigma}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^\mu_\mu}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\mu}\right)....\quad (671)$$

There isn't index ν in the second term of the right side of (662) ~ (671) at all here. Therefore, I rewrite dummy index σ of (662) ~ (671) in dummy index ν and can get

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}-2\frac{\partial}{\partial x^\mu}\left(A^\nu \frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^\nu_\nu}{\partial x^\nu}\right)...,\quad (672)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}-2\frac{\partial}{\partial x^\mu}\left(A^\nu \frac{1}{2}\left(\frac{\partial g^\mu_\nu}{\partial x^\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^\nu_\nu}{\partial x_\mu}\right)...,\quad (673)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}-2\frac{\partial}{\partial x^\mu}\left(A^\nu \frac{1}{2}\left(\frac{\partial g^\mu_\mu}{\partial x^\nu}\right)\right)\frac{1}{2}\left(\frac{\partial g^\nu_\nu}{\partial x_\mu}\right)...,\quad (674)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}-2\frac{\partial}{\partial x^\nu}\left(A^\nu \frac{1}{2}\left(\frac{\partial g^\mu_\mu}{\partial x^\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^\nu_\nu}{\partial x_\mu}\right)...,\quad (675)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}-2\frac{\partial}{\partial x^\mu}\left(A^\nu \frac{1}{2}\left(\frac{\partial g^\mu_\nu}{\partial x^\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^\nu}\right)...,\quad (676)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}-2\frac{\partial}{\partial x^\mu}\left(A^\nu \frac{1}{2}\left(\frac{\partial g^\mu_\mu}{\partial x^\nu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^\nu}\right)...,\quad (677)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}-2\frac{\partial}{\partial x^\nu}\left(A^\nu \frac{1}{2}\left(\frac{\partial g^\mu_\mu}{\partial x^\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^\nu}\right)...,\quad (678)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}-2\frac{\partial}{\partial x^\mu}\left(A^\nu \frac{1}{2}\left(\frac{\partial g^\mu_\nu}{\partial x^\nu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^\mu}\right)...,\quad (679)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}-2\frac{\partial}{\partial x^\nu}\left(A^\nu \frac{1}{2}\left(\frac{\partial g^\mu_\nu}{\partial x^\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^\mu}\right)...,\quad (680)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}-2\frac{\partial}{\partial x^\nu}\left(A^\nu \frac{1}{2}\left(\frac{\partial g^\mu_\mu}{\partial x^\nu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^\mu}\right)....\quad (681)$$

Furthermore, I rewrite index ν of the coordinate systems of (142) ~ (151) by Definision 3, Definision 2 and get

$$-(A^{\mu}_{;\mu})^{;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x_{\mu}}+2\frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\mu}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\sigma}}\right)..., \quad (682)$$

$$-(A^{\mu}_{;\mu})^{;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x_{\mu}}+2\frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)..., \quad (683)$$

$$-(A^{\mu}_{;\mu})^{;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x_{\mu}}+2\frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)..., \quad (684)$$

$$-(A^{\mu}_{;\mu})^{;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x_{\mu}}+2\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)..., \quad (685)$$

$$-(A^{\mu}_{;\mu})^{;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x_{\mu}}+2\frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)..., \quad (686)$$

$$-(A^{\mu}_{;\mu})^{;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x_{\mu}}+2\frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)..., \quad (687)$$

$$-(A^{\mu}_{;\mu})^{;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x_{\mu}}+2\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)..., \quad (688)$$

$$-(A^{\mu}_{;\mu})^{;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x_{\mu}}+2\frac{\partial}{\partial x_{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x_{\mu}}\right)..., \quad (689)$$

$$-(A^{\mu}_{;\mu})^{;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x_{\mu}}+2\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x_{\mu}}\right)..., \quad (690)$$

$$-(A^{\mu}_{;\mu})^{;\mu;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x_{\mu}}+2\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x_{\mu}}\right).... \quad (691)$$

Because two same contravariant index existed in $-(A^{\mu}_{;\mu})^{;\mu;\mu}$ of (682) ~ (691), I decided not to handle (682) ~ (691). Furthermore, I rewrite index ν of the coordinate systems of (142) ~ (151) by Definision 3, Definision 2 and get

$$-(A^{\mu;\mu;\mu})_{;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x^{\mu}}+2\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\sigma}}\right)..., \quad (692)$$

$$-(A^{\mu;\mu;\mu})_{;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x^{\mu}}+2\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu}_{\sigma}}{\partial x_{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_{\mu}}\right)..., \quad (693)$$

$$-(A^{\mu;\mu;\mu})_{;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x^{\mu}}+2\frac{\partial}{\partial x^{\mu}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\sigma}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_{\mu}}\right)..., \quad (694)$$

$$-(A^{\mu;\mu;\mu})_{;\mu}=-\frac{\partial^3 A^{\mu}}{\partial x_{\mu}\partial x_{\mu}\partial x^{\mu}}+2\frac{\partial}{\partial x^{\sigma}}\left(A^{\sigma}\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^{\mu}}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_{\mu}}\right)..., \quad (695)$$

$$-(A^{\mu;\mu;\mu})_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}+2\frac{\partial}{\partial x^\mu}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x_\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)..., \quad (696)$$

$$-(A^{\mu;\mu;\mu})_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}+2\frac{\partial}{\partial x^\mu}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)..., \quad (697)$$

$$-(A^{\mu;\mu;\mu})_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}+2\frac{\partial}{\partial x^\sigma}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)..., \quad (698)$$

$$-(A^{\mu;\mu;\mu})_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}+2\frac{\partial}{\partial x^\mu}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_\mu}\right)..., \quad (699)$$

$$-(A^{\mu;\mu;\mu})_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}+2\frac{\partial}{\partial x^\sigma}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x^\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_\mu}\right)..., \quad (700)$$

$$-(A^{\mu;\mu;\mu})_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}+2\frac{\partial}{\partial x^\sigma}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^\mu_\mu}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_\mu}\right).... \quad (701)$$

Because two same contravariant index existed in $-(A^{\mu;\mu;\mu})_{;\mu}$ of (692) ~ (701), I decided not to handle (692) ~ (701). Furthermore, I rewrite index ν of the coordinate systems of (142) ~ (151) by Definision 3, Definision 2 and get

$$-(A^{\mu;\mu})^{;\mu}_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}+2\frac{\partial}{\partial x_\mu}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\sigma}\right)..., \quad (702)$$

$$-(A^{\mu;\mu})^{;\mu}_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}+2\frac{\partial}{\partial x_\mu}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x^\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x_\mu}\right)..., \quad (703)$$

$$-(A^{\mu;\mu})^{;\mu}_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}+2\frac{\partial}{\partial x_\mu}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^\mu_\mu}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x_\mu}\right)..., \quad (704)$$

$$-(A^{\mu;\mu})^{;\mu}_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}+2\frac{\partial}{\partial x^\sigma}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^\mu_\mu}{\partial x_\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x_\mu}\right)..., \quad (705)$$

$$-(A^{\mu;\mu})^{;\mu}_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}+2\frac{\partial}{\partial x_\mu}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x^\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)..., \quad (706)$$

$$-(A^{\mu;\mu})^{;\mu}_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}+2\frac{\partial}{\partial x_\mu}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^\mu_\mu}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)..., \quad (707)$$

$$-(A^{\mu;\mu})^{;\mu}_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}+2\frac{\partial}{\partial x^\sigma}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^\mu_\mu}{\partial x_\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)..., \quad (708)$$

$$-(A^{\mu;\mu})^{;\mu}_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}+2\frac{\partial}{\partial x_\mu}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\mu}\right)..., \quad (709)$$

$$-(A^{\mu;\mu})^{;\mu}_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}+2\frac{\partial}{\partial x^\sigma}\left(A^\sigma \frac{1}{2}\left(\frac{\partial g^\mu_\sigma}{\partial x_\mu}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\mu}\right)..., \quad (710)$$

$$-(A^{\mu;\mu})^{;\mu}_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}+2\frac{\partial}{\partial x^\sigma}\left(A^\sigma\frac{1}{2}\left(\frac{\partial g^{\mu\mu}}{\partial x^\sigma}\right)\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\mu}\right).... \quad (711)$$

Because two same contravariant index existed in $-(A^{\mu;\mu})^{;\mu}_{;\mu}$ of (702) ~ (711), I decided not to handle (702) ~ (711). Similarly, I rewrite index ν of the coordinate systems of (152) ~ (161) by Definision 3 and get

$$-A^\mu_{;\mu;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x^\mu}-2\frac{\partial A^\mu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\sigma}\right)..., \quad (712)$$

$$-A^\mu_{;\mu;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x^\mu}-2\frac{\partial A^\mu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\mu}\right)..., \quad (713)$$

$$-A^\mu_{;\mu;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x^\mu}-2\frac{\partial A^\mu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\mu}\right)..., \quad (714)$$

$$-A^\mu_{;\mu;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x^\mu}-2\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\mu}\right)..., \quad (715)$$

$$-A^\mu_{;\mu;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x^\mu}-2\frac{\partial A^\mu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^\sigma}\right)..., \quad (716)$$

$$-A^\mu_{;\mu;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x^\mu}-2\frac{\partial A^\mu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^\sigma}\right)..., \quad (717)$$

$$-A^\mu_{;\mu;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x^\mu}-2\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^\sigma}\right)..., \quad (718)$$

$$-A^\mu_{;\mu;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x^\mu}-2\frac{\partial A^\mu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^\mu}\right)..., \quad (719)$$

$$-A^\mu_{;\mu;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x^\mu}-2\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^\mu}\right)..., \quad (720)$$

$$-A^\mu_{;\mu;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x^\mu}-2\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^\mu}\right).... \quad (721)$$

Because two same covariant index existed in $-A^\mu_{;\mu;\mu;\mu}$ of (712) ~ (721), I decided not to handle (712) ~ (721). Furthermore, I rewrite index ν of the coordinate systems of (152) ~ (161) by Definision 2 and get

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x_\mu}+2\frac{\partial A^\mu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\sigma}\right)..., \quad (722)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x_\mu}+2\frac{\partial A^\mu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x_\mu}\right)..., \quad (723)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x_\mu}+2\frac{\partial A^\mu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x_\mu}\right)..., \quad (724)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x_\mu}+2\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x_\mu}\right)..., \quad (725)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x_\mu}+2\frac{\partial A^\mu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)..., \quad (726)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x_\mu}+2\frac{\partial A^\mu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)..., \quad (727)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x_\mu}+2\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)..., \quad (728)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x_\mu}+2\frac{\partial A^\mu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_\mu}\right)..., \quad (729)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x_\mu}+2\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_\mu}\right)..., \quad (730)$$

$$A^{\mu;\mu;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x_\mu}+2\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_\mu}\right).... \quad (731)$$

Because two same contravariant index existed in $A^{\mu;\mu;\mu;\mu}$ of (722) ~ (731), I decided not to handle (722) ~ (731). Furthermore, I rewrite index ν of the coordinate systems of (152) ~ (161) by Definision 3, Definision 2 and get

$$(A^\mu_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x_\mu}+2\frac{\partial A^\mu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\sigma}\right)..., \quad (732)$$

$$(A^\mu_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x_\mu}+2\frac{\partial A^\mu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\right)..., \quad (733)$$

$$(A^\mu_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x_\mu}+2\frac{\partial A^\mu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\right)..., \quad (734)$$

$$(A^\mu_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x_\mu}+2\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\right)..., \quad (735)$$

$$(A^\mu_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x_\mu}+2\frac{\partial A^\mu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\sigma}\right)..., \quad (736)$$

$$(A^\mu_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x_\mu}+2\frac{\partial A^\mu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\sigma}\right)..., \quad (737)$$

$$(A^\mu_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x_\mu}+2\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\sigma}\right)..., \quad (738)$$

$$(A^\mu_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x_\mu}+2\frac{\partial A^\mu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\mu}\right)..., \quad (739)$$

$$(A^\mu_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x_\mu}+2\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\mu}\right)..., \quad (740)$$

$$(A^\mu_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x_\mu}+2\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\mu}\right).... \quad (741)$$

There isn't index ν in the second term of the right side of (732) ~ (741) at all here. Therefore, I rewrite dummy index σ of (732) ~ (741) in dummy index ν and can get

$$(A^{\mu}_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+2\frac{\partial A^{\mu}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\nu}}\right)\dots, \tag{742}$$

$$(A^{\mu}_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+2\frac{\partial A^{\mu}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\right)\dots, \tag{743}$$

$$(A^{\mu}_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+2\frac{\partial A^{\mu}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\right)\dots, \tag{744}$$

$$(A^{\mu}_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+2\frac{\partial A^{\mu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x_{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\right)\dots, \tag{745}$$

$$(A^{\mu}_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+2\frac{\partial A^{\mu}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\right)\dots, \tag{746}$$

$$(A^{\mu}_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+2\frac{\partial A^{\mu}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\right)\dots, \tag{747}$$

$$(A^{\mu}_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+2\frac{\partial A^{\mu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x_{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\right)\dots, \tag{748}$$

$$(A^{\mu}_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+2\frac{\partial A^{\mu}}{\partial x_{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu}}\right)\dots, \tag{749}$$

$$(A^{\mu}_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+2\frac{\partial A^{\mu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x_{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu}}\right)\dots, \tag{750}$$

$$(A^{\mu}_{;\mu;\mu})^{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x^{\mu}\partial x_{\mu}}+2\frac{\partial A^{\mu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\mu}}\right)\dots. \tag{751}$$

Furthermore, I rewrite index ν of the coordinate systems of (152) ~ (161) by Definision 3, Definision 2 and get

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+2\frac{\partial A^{\mu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x_{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\sigma}}\right)\dots, \tag{752}$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+2\frac{\partial A^{\mu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)\dots, \tag{753}$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+2\frac{\partial A^{\mu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)\dots, \tag{754}$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+2\frac{\partial A^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)\dots, \tag{755}$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+2\frac{\partial A^{\mu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x_{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)\dots, \tag{756}$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+2\frac{\partial A^{\mu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)...,\quad(757)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+2\frac{\partial A^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)...,\quad(758)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+2\frac{\partial A^{\mu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x_{\mu}}\right)...,\quad(759)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+2\frac{\partial A^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\sigma}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x_{\mu}}\right)...,\quad(760)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+2\frac{\partial A^{\mu}}{\partial x^{\sigma}}\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x^{\sigma}}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma}_{\mu}}{\partial x_{\mu}}\right)....\quad(761)$$

There isn't index ν in the second term of the right side of (752) ~ (761) at all here. Therefore, I rewrite dummy index σ of (752) ~ (761) in dummy index ν and can get

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+2\frac{\partial A^{\mu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x_{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\nu}}\right)...,\quad(762)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+2\frac{\partial A^{\mu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x_{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\right)...,\quad(763)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+2\frac{\partial A^{\mu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\right)...,\quad(764)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+2\frac{\partial A^{\mu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\right)...,\quad(765)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+2\frac{\partial A^{\mu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x_{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\right)...,\quad(766)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+2\frac{\partial A^{\mu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\right)...,\quad(767)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+2\frac{\partial A^{\mu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\right)...,\quad(768)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+2\frac{\partial A^{\mu}}{\partial x^{\mu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x_{\mu}}\right)...,\quad(769)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+2\frac{\partial A^{\mu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\nu}}{\partial x^{\mu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x_{\mu}}\right)...,\quad(770)$$

$$(A^{\mu}_{;\mu})^{;\mu}_{;\mu}=\frac{\partial^3 A^{\mu}}{\partial x^{\mu}\partial x_{\mu}\partial x^{\mu}}+2\frac{\partial A^{\mu}}{\partial x^{\nu}}\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x^{\nu}}\right)\frac{1}{2}\left(\frac{\partial g^{\nu}_{\mu}}{\partial x_{\mu}}\right)....\quad(771)$$

Furthermore, I rewrite index ν of the coordinate systems of (152) ~ (161) by Definision 3,Definision 2 and get

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}+2\frac{\partial A^\mu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\sigma}\right)..., \quad (772)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}+2\frac{\partial A^\mu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x_\mu}\right)..., \quad (773)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}+2\frac{\partial A^\mu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x_\mu}\right)..., \quad (774)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}+2\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x_\mu}\right)..., \quad (775)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}+2\frac{\partial A^\mu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)..., \quad (776)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}+2\frac{\partial A^\mu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)..., \quad (777)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}+2\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)..., \quad (778)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}+2\frac{\partial A^\mu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\mu}\right)..., \quad (779)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}+2\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\mu}\right)..., \quad (780)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}+2\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\mu}\right).... \quad (781)$$

There isn't index ν in the second term of the right side of (772) ~ (781) at all here. Therefore, I rewrite dummy index σ of (772) ~ (781) in dummy index ν and can get

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}+2\frac{\partial A^\mu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g_\nu^\nu}{\partial x^\nu}\right)..., \quad (782)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}+2\frac{\partial A^\mu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g_\nu^\nu}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g_\nu^\nu}{\partial x_\mu}\right)..., \quad (783)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}+2\frac{\partial A^\mu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g_\mu^\nu}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g_\nu^\nu}{\partial x_\mu}\right)..., \quad (784)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}+2\frac{\partial A^\mu}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g_\mu^\nu}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g_\nu^\nu}{\partial x_\mu}\right)..., \quad (785)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}+2\frac{\partial A^\mu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g_\nu^\nu}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^\nu}\right)..., \quad (786)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}+2\frac{\partial A^\mu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g_\mu^\nu}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^\nu}\right)..., \quad (787)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}+2\frac{\partial A^\mu}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g^\nu_\mu}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^\nu}\right)..., \quad (788)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}+2\frac{\partial A^\mu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^\nu_\nu}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^\mu}\right)..., \quad (789)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}+2\frac{\partial A^\mu}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g^\nu_\nu}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^\mu}\right)..., \quad (790)$$

$$(A^{\mu;\mu})_{;\mu;\mu}=\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu}+2\frac{\partial A^\mu}{\partial x^\nu}\frac{1}{2}\left(\frac{\partial g^\nu_\mu}{\partial x^\nu}\right)\frac{1}{2}\left(\frac{\partial g^{\nu\mu}}{\partial x^\mu}\right).... \quad (791)$$

Furthermore, I rewrite index ν of the coordinate systems of (152) ~ (161) by Definision 3, Definision 2 and get

$$-(A^\mu_{;\mu})^{;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu}-2\frac{\partial A^\mu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\sigma}\right)..., \quad (792)$$

$$-(A^\mu_{;\mu})^{;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu}-2\frac{\partial A^\mu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\mu}\right)..., \quad (793)$$

$$-(A^\mu_{;\mu})^{;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu}-2\frac{\partial A^\mu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\mu}\right)..., \quad (794)$$

$$-(A^\mu_{;\mu})^{;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu}-2\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\mu}\right)..., \quad (795)$$

$$-(A^\mu_{;\mu})^{;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu}-2\frac{\partial A^\mu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^\sigma}\right)..., \quad (796)$$

$$-(A^\mu_{;\mu})^{;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu}-2\frac{\partial A^\mu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^\sigma}\right)..., \quad (797)$$

$$-(A^\mu_{;\mu})^{;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu}-2\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x^\sigma}\right)..., \quad (798)$$

$$-(A^\mu_{;\mu})^{;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu}-2\frac{\partial A^\mu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x_\mu}\right)..., \quad (799)$$

$$-(A^\mu_{;\mu})^{;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu}-2\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x_\mu}\right)..., \quad (800)$$

$$-(A^\mu_{;\mu})^{;\mu;\mu}=-\frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x_\mu}-2\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\mu}{\partial x_\mu}\right).... \quad (801)$$

Because two same contravariant index existed in $-(A^\mu_{;\mu})^{;\mu;\mu}$ of (792) ~ (801), I decided not to handle (792) ~ (801). Furthermore, I rewrite index ν of the coordinate systems of (152) ~ (161) by Definision 3, Definision 2 and get

$$-(A^{\mu;\mu;\mu})_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}-2\frac{\partial A^\mu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\sigma}\right)..., \quad (802)$$

$$-(A^{\mu;\mu;\mu})_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}-2\frac{\partial A^\mu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x_\mu}\right)..., \quad (803)$$

$$-(A^{\mu;\mu;\mu})_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}-2\frac{\partial A^\mu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x_\mu}\right)..., \quad (804)$$

$$-(A^{\mu;\mu;\mu})_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}-2\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x_\mu}\right)..., \quad (805)$$

$$-(A^{\mu;\mu;\mu})_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}-2\frac{\partial A^\mu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)..., \quad (806)$$

$$-(A^{\mu;\mu;\mu})_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}-2\frac{\partial A^\mu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)..., \quad (807)$$

$$-(A^{\mu;\mu;\mu})_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}-2\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)..., \quad (808)$$

$$-(A^{\mu;\mu;\mu})_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}-2\frac{\partial A^\mu}{\partial x^\mu}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_\mu}\right)..., \quad (809)$$

$$-(A^{\mu;\mu;\mu})_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}-2\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_\mu}\right)..., \quad (810)$$

$$-(A^{\mu;\mu;\mu})_{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x_\mu \partial x^\mu}-2\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x_\mu}\right).... \quad (811)$$

Because two same contravariant index existed in $-(A^{\mu;\mu;\mu})_{;\mu}$ of (802) ~ (811), I decided not to handle (802) ~ (811). Furthermore, I rewrite index ν of the coordinate systems of (152) ~ (161) by Definision 3, Definision 2 and get

$$-(A^{\mu;\mu})_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}-2\frac{\partial A^\mu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\sigma}\right)..., \quad (812)$$

$$-(A^{\mu;\mu})_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}-2\frac{\partial A^\mu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x_\mu}\right)..., \quad (813)$$

$$-(A^{\mu;\mu})_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}-2\frac{\partial A^\mu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x_\mu}\right)..., \quad (814)$$

$$-(A^{\mu;\mu})_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}-2\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x_\mu}\right)..., \quad (815)$$

$$-(A^{\mu;\mu})_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}-2\frac{\partial A^\mu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g_\sigma^\sigma}{\partial x^\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)..., \quad (816)$$

$$-(A^{\mu;\mu})_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}-2\frac{\partial A^\mu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)..., \quad (817)$$

$$-(A^{\mu;\mu})_{;\mu}^{;\mu}=-\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu}-2\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g_\mu^\sigma}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)..., \quad (818)$$

$$-(A^{\mu;\mu})^{;\mu}_{;\mu} = -\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu} - 2\frac{\partial A^\mu}{\partial x_\mu}\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\mu}\right)..., \quad (819)$$

$$-(A^{\mu;\mu})^{;\mu}_{;\mu} = -\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu} - 2\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^\sigma_\sigma}{\partial x_\mu}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\mu}\right)..., \quad (820)$$

$$-(A^{\mu;\mu})^{;\mu}_{;\mu} = -\frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x_\mu} - 2\frac{\partial A^\mu}{\partial x^\sigma}\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\sigma}\right)\frac{1}{2}\left(\frac{\partial g^{\sigma\mu}}{\partial x^\mu}\right).... \quad (821)$$

Because two same contravariant index existed in $-(A^{\mu;\mu})^{;\mu}_{;\mu}$ of (812) ~ (821), I decided not to handle (812) ~ (821). And I get

$$(A^\mu_{;\mu;\mu})^{;\mu} = \frac{\partial^3 A^\mu}{\partial x^\mu \partial x^\mu \partial x_\mu} \quad (822)$$

from (174), (218), (262), (306), (368) ~ (377), (478) ~ (487), (570), (632) ~ (641), (742) ~ (751) in consideration of Definision 5. And I get

$$(A^\mu_{;\mu})^{;\mu}_{;\mu} = \frac{\partial^3 A^\mu}{\partial x^\mu \partial x_\mu \partial x^\mu} \quad (823)$$

from (182), (226), (270), (314), (388) ~ (397), (498) ~ (507), (578), (652) ~ (661), (762) ~ (771) in consideration of Definision 5. And I get

$$(A^{\mu;\mu})_{;\mu;\mu} = \frac{\partial^3 A^\mu}{\partial x_\mu \partial x^\mu \partial x^\mu} \quad (824)$$

from (190), (234), (278), (322), (408) ~ (417), (518) ~ (527), (586), (672) ~ (681), (782) ~ (791) in consideration of Definision 5. In addition,

$$A^\mu_{;\nu;\nu;\nu} = \frac{\partial^3 A^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu} \quad (825)$$

can rewrite (822), (823), (824) using Definision 2, Definision 3 because the second term doesn't exist in the right side of (822), (823), (824). Because (825) doesn't change the form of the equation even if all coordinate systems satisfies Binary Law, I get the conclusion that (825) satisfies Binary Law. In other words, (825) is an equation of the tensor satisfying Binary Law. Because (825) is an equation provided from Definision 8, (825) is the equation that was rewritten so that Definision8 is an equation of the tensor satisfying Binary Law.

– End Proof –

6. Property of Tensor Satisfying Binary Law about the Third-Order Covariant Derivative of the Contravariant Vector

Proposition 7 *If* $\overline{x^\mu} \neq x^\mu, \overline{x^\nu} \neq x^\nu, \overline{x^\mu} = x^\nu, \overline{x^\nu} = x^\mu$ is established, $\left\{x^\mu\right\}_M = \sin\sqrt{M}x^\nu$ is established.

Proof: I change in A^μ to x^μ in Proposition 6 and get

$$x^\mu_{;\nu;\nu;\nu} = \frac{\partial^3 x^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu} \quad . \quad (826)$$

I express the right side of (826) in

$$\frac{\partial^3 x^\mu}{\partial x^\nu \partial x^\nu \partial x^\nu} = M. \quad (827)$$

I get

$$\begin{array}{l}\frac{\partial^3 x^1}{\partial \dot{x}^1 \partial \dot{x}^1 \partial \dot{x}^1} = M, \frac{\partial^3 x^1}{\partial \dot{x}^2 \partial \dot{x}^2 \partial \dot{x}^2} = M, \\ \frac{\partial^3 x^2}{\partial \dot{x}^1 \partial \dot{x}^1 \partial \dot{x}^1} = M, \frac{\partial^3 x^2}{\partial \dot{x}^2 \partial \dot{x}^2 \partial \dot{x}^2} = M\end{array} \quad (828)$$

from (827) if a dimensional number is 2. I get

$$\begin{array}{l}\int \frac{\partial^3 x^1}{\partial \dot{x}^1 \partial \dot{x}^1 \partial \dot{x}^1} d\dot{x}^1 \int M d\dot{x}^1, \int \frac{\partial^3 x^1}{\partial \dot{x}^2 \partial \dot{x}^2 \partial \dot{x}^2} d\dot{x}^2 = \int M d\dot{x}^2, \\ \int \frac{\partial^3 x^2}{\partial \dot{x}^1 \partial \dot{x}^1 \partial \dot{x}^1} d\dot{x}^1 \int M d\dot{x}^1, \int \frac{\partial^3 x^2}{\partial \dot{x}^2 \partial \dot{x}^2 \partial \dot{x}^2} d\dot{x}^2 = \int M d\dot{x}^2\end{array} \quad (829)$$

from (828). And I get

$$\begin{array}{l}\frac{\partial^2 x^1}{\partial \dot{x}^1 \partial \dot{x}^1} = M\dot{x}^1, \frac{\partial^2 x^1}{\partial \dot{x}^2 \partial \dot{x}^2} = M\dot{x}^2, \\ \frac{\partial^2 x^2}{\partial \dot{x}^1 \partial \dot{x}^1} = M\dot{x}^1, \frac{\partial^2 x^2}{\partial \dot{x}^2 \partial \dot{x}^2} = M\dot{x}^2\end{array} \quad (830)$$

from (829). I get

$$\frac{\partial^2 x^\mu}{\partial x^\nu \partial x^\nu} = M x^\nu \quad (831)$$

from (830). I get

$$\frac{\partial^2 x^\mu}{\partial x^\nu \partial x^\nu} = -M x^\mu \quad (832)$$

from (831), Definision 3. And I get

$$\begin{array}{l}\frac{\partial^2 x^1}{\partial \dot{x}^1 \partial \dot{x}^1} = -Mx^1, \frac{\partial^2 x^1}{\partial \dot{x}^2 \partial \dot{x}^2} = -Mx^1, \\ \frac{\partial^2 x^2}{\partial \dot{x}^1 \partial \dot{x}^1} = -Mx^2, \frac{\partial^2 x^2}{\partial \dot{x}^2 \partial \dot{x}^2} = -Mx^2\end{array} \quad (833)$$

from (832). I get

$$\begin{array}{l}x^1 = \sin\sqrt{M}\ \dot{x}^1, x^1 = \sin\sqrt{M}\ \dot{x}^2, \\ x^2 = \sin\sqrt{M}\ \dot{x}^1, x^2 = \sin\sqrt{M}\ \dot{x}^2\end{array} \quad (834)$$

from (833). And I get

$$\left\{x^{\mu}\right\}_{M}=\sin\sqrt{M}\,x^{\nu} \quad (835)$$

from (834), Definision 12.

– End Proof –

7. Discussion

About Proposition 7

If $M>0$ is established, (833) doesn't change. On the other hand, (833) becomes

$\frac{\partial^2 x^1}{\partial \dot{x}^1 \partial \dot{x}^1}=0, \frac{\partial^2 x^1}{\partial \dot{x}^2 \partial \dot{x}^2}=0, \frac{\partial^2 x^2}{\partial \dot{x}^1 \partial \dot{x}^1}=0, \frac{\partial^2 x^2}{\partial \dot{x}^2 \partial \dot{x}^2}=0$ if M=0 is established. Furthermore, (833) becomes

$\frac{\partial^2 x^1}{\partial \dot{x}^1 \partial \dot{x}^1}=Mx^1, \frac{\partial^2 x^1}{\partial \dot{x}^2 \partial \dot{x}^2}=Mx^1, \frac{\partial^2 x^2}{\partial \dot{x}^1 \partial \dot{x}^1}=Mx^2, \frac{\partial^2 x^2}{\partial \dot{x}^2 \partial \dot{x}^2}=Mx^2$ If $M<0$ is established. Thus, (833) is

Established when $M>0$ is established. In other words, $\left\{x^{\mu}\right\}_{M}$ can have a property of waving when $M>0$ is established.

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Citation: Koji Ichidayama (2018) Tensor Satisfying Binary Law about the First, Second, Third-Order Covariant Derivative of the Contravariant Vector. SF J Nuclear Science 2:3.