The Role of Magnetic Field in Astrophysics and its Application

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Abstract

The role and influence of gravitation on phenomena of astrophysics is immeasurable. It is well known that gravitation is generated by curvature of space. The curvature of the space is generated not only by object mass but also by magnetic field. In this paper, we introduce three applications using spatial curvature induced by magnetic field, that is, another collimation mechanism of astrophysical jet, gravitational wave generation, advanced space propulsion by space drive.

Keywords

Magnetic Field; Astrophysical Jet; Accretion Disk; Black Hole; Collimation Mechanism; Continuum; Curvature; Gravitational Wave; Propulsion

1. Introduction

At the present day, owing to the development of observational technology, appearance of astrophysical phenomena is changing. Here, astrophysical phenomena refer mainly accretion disk and astrophysical jet around black holes. Accretion disk is rotating gaseous disk with accretion flow, which form around gravitating object, such as white dwarfs, neutron stars, and black holes. It is believed that accretion disk causes the various active phenomena in the universe: star formation, high energy radiation, astrophysical jet, and so on.

It should be noted; these stars such as white dwarfs, neutron stars, and black holes have a strong magnetic field ($10^8$ Tesla-$10^{11}$ Tesla). Matter falling onto an accretion disk around black hole is ejected in narrow jet moving at close to the speed of light like an accelerator. Entity of the astrophysical jet is a jet of plasma gas from the active galactic nucleus (accretion disk in there).

It is said that such astrophysical jet is held together by strong magnetic field tendrils, while the jet’s light is created by particles revolving around these thin magnetic field lines. Furthermore, since the system of black hole and accretion disk is like a gravitational power plants, the energy of the heat and the light are produced by the release of gravitational energy.

Figure 1 shows astrophysical jet and accretion disk around black hole.

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Although the system of accretion disk and astrophysical jet around black holes are currently left many unresolved issues, the elucidation of these mechanisms and principles that are common to the entire universe may provide a new application. Especially, the breaking of magnetic field lines and magnetic field reconnection are possible to produce many kinds of charged particles such as electron-positron pairs. Generally, in a high-temperature plasma, electron-positron pairs are readily formed by collisions between the high energy protons, electrons, photons. Since the dynamics of the accretion disk has been decided by a magnetic field, it is important to solve the dynamics of the magnetic field.

In this way, it is assumed that the magnetic field plays an important role at the core of the astrophysical phenomenon.

Now, gravitation is well known as an essential element in such astrophysical phenomena and cosmology.

In General Relativity, the effects of gravitation are ascribed to spacetime curvature. Einstein proposed that spacetime is curved by matter, and that free-falling objects are moving along locally straight paths in curved spacetime.

However, spacetime curvature is generated not only by matter but also by electromagnetic field.

From General Relativity, the major component of curvature of space (here in after referred to as the major component of spatial curvature) $R^{00}$ can be produced by not only mass density but also magnetic field B as follows (See Appendix A: Curvature Control by Magnetic Field):

$$R^{00} = \frac{4\pi G}{\mu_0 c^4} \cdot \frac{B^2}{8.2 \times 10^{-38}} \cdot B^2 \text{ (B in Tesla)}$$

(1)

where we let $\mu_0 = 4\pi \times 10^{-7} (H/m)$, $\varepsilon_0 = 1/(36\pi) \times 10^{-9} (F/m)$, $c = 3 \times 10^8 (m/s)$

$G = 6.672 \times 10^{-11} \left( N \cdot m^2 / Kg^2 \right)$, $B$ is a magnetic field in Tesla and $R^{00}$ is a major component of spatial curvature $\left(1 / m^2 \right)$
Eq.(1) indicates that the major component of spatial curvature can be controlled by magnetic field B.

The relationship between curvature and magnetic field was derived by Minami and introduced it in 16th International Symposium on Space Technology and Science (1988) [1]. Eq.(1) is derived from general method using gravitational field equation. On the other hand, Levi-Civita also investigated the gravitational field produced by a homogeneous electric or magnetic field, which was expressed by Pauli (See Appendix A: Curvature Control by Magnetic Field) [2].

General Relativity is based on Riemannian geometry. Riemannian geometry is a geometry that deals with a curved Riemann space, therefore Riemann curvature tensor is the principal quantity. All components of Riemann curvature tensor are zero for flat space (strictly speaking, only 20 independent components of Riemann curvature tensor \( R_{\mu\nu\rho\sigma} \) are zero) and non-zero for curved space. If an only non-zero component of Riemann curvature tensor exists, the space is not flat space but curved space. Although Ricci tensor \( R_{\mu\nu} \) has 10 independent components, the major component is the case of \( \mu = \nu = 0 \), i.e., \( R^{00} \). Therefore, the major curvature of Ricci tensor \( R^{00} \), that is, major component of spatial curvature plays a significant role.

In the following, the application to astrophysics concerning the spatial curvature generated by magnetic field will be respectively introduced. They are “Application for Acceleration and Collimation Mechanism of Astrophysical Jet”, “Application for Gravitational Wave Generation”, and “Application for Space Propulsion System”.

2. Application for Acceleration and Collimation Mechanism of Astrophysical Jet
2.1 Overview of Collimation and Acceleration Mechanism of Astrophysical Jet

The astrophysical jet is a narrow jetted plasma jet at high speed (100 km/s to near the speed of light) that emits in both directions vertically from accretion disk around the compact central object such as a neutron star or black hole. Its length is an enormous, long and narrow jet reaching from 1 light year-10 light years-1 million light years. A jet propagating at a speed close to the speed of light is called a relativistic jet (See Figure 1).

The acceleration mechanism of the astrophysical jet and the collimation mechanism narrowing down to a long distance have been examined so far. They are due to thermal gas pressure, light radiation pressure, and magnetic field pressure. Currently, Radiative Acceleration model accelerated by the radiation field of the accretion disk and Magnetic Acceleration model accelerated by magnetic field penetrating the accretion disk are representative models. The high velocity, highly collimated gas streams - jets - raise two major problems, namely, how the jet material is accelerated, and how it is collimated. Figure 2 shows Formation of Astrophysical Jet wound by twisting magnetic field lines.

It is a collimation problem of how to narrow the jet narrowly, and the model of the jet acceleration mechanism is required to solve this collimation problem at the same time as well as acceleration. At the present time, the magnetic force model (magnetic centrifugal force and magnetic pressure) is regarded as the most dominant theory which solves the two problems of jet acceleration and collimation at the same time. That is, the accretion disk generates a helical magnetic field by twisting the magnetic field lines, accelerates by magnetic force, and narrows the jet by magnetic tension (pinch). The self-pinching force of magnetic field twisted by the rotation occurs naturally as a force to collimate the jet thinly (Figure 2) [3-11].

If the magnetic field lines are in the jet, there is a possibility that a strong magnetic field region is locally generated due to local turbulence and shock waves in the plasma.

Figure 3 shows Interstellar magnetic field lines. Since the plasma being highly conductive, it will be expected that the interstellar magnetic field lines will become frozen into that plasma which rotates within accretion disk, and as the accretion disk rotates it will drag and twist those magnetic field lines, pulling them together.

Figure 4 shows a toroidal field dominated jet is launched by magnetic pressure. As the magnetic field penetrating the accretion disk is twisted, the energy of the magnetic field is accumulated, and at the same time that it propagates along the magnetic field lines, the jet ejects from the accretion disk, and not only the magnetic centrifugal force but also magnetic pressure contributes to acceleration of jet.
Figure 2: Formation of Astrophysical Jet wound by twisting magnetic field lines (https://search.yahoo.co.jp/image/search)

Figure 3: Interstellar magnetic field lines will become frozen into that plasma which rotates within accretion disk

Figure 4: KatoY, Mineshige, Shibata (2004); 3D sim. (ApJ). This toroidal field dominated jet is launched by magnetic pressure (similar to Shibata and Uchida 1985, Turner et al. 1999, Kudoh et al. 2002), and is also Similar to “magnetic tower” of Lynden-Bell (1996)
When the magnetic field is twisted in the direction of rotation by the actuation rotation of plasma material, the twisted magnetic field acts like a spring to accelerate the plasma material further upward.

In other words, it is acceleration by magnetic pressure. If the jet is magnetically accelerated, the jet is expected to have a twisted helical magnetic field (Figure 2). Further more, the twisted magnetic field acts like a rubber string, and the force of the rubber band shrinks (magnetic pinch) so that the flow of the plasma substance is directed in the rotation axis direction. This is the collimation of the jet by magnetic field. Collimation also occurs voluntarily in addition to acceleration in a model where a jet is driven from an accretion disk by magnetic field and rotation.

Even if the magnetic field penetrating the accretion disk is very weak, the rotation of the accretion disk causes the magnetic field to twist and increase more and more, and the energy is stored in the magnetic field to the same extent as the rotational energy of the accretion disk. Even in the case of a local magnetic field in the disk instead of the global magnetic field, the magnetic field is twisted in the disk, so that magnetic pressure is generated and it is possible to accelerate the jet.

The energy of the magnetic field is increased by compressing the gas or stretching the magnetic field lines due to the plasma gas. The phenomenon in which energy stored in the form of a magnetic field is released locally and in large quantities in a short time is well known for solar flares. The acceleration mechanism for these jets may be similar to the magnetic reconnection processes observed in the Earth’s magnetosphere and the solar wind.

As described above, at the present time, the magnetic force model (magnetic centrifugal force and magnetic pressure) is regarded as the most dominant theory which solves the two problems of jet acceleration and collimation at the same time.

Next we introduce another Collimation Mechanism of Astrophysical Jet.

2.2 Another Collimation Mechanism due to the Pressure Field Induced by Spatial Curvature

The principle of this idea is derived from General Relativity and the theory of continuum mechanics. We assume that the so-called “vacuum” of space acts as an infinite elastic body like rubber. The curvature of space plays a significant role. Furthermore, the major component of curvature of space can be produced by not only mass density but also magnetic field.

On the supposition that space is an infinite continuum, continuum mechanics can be applied to the so-called “vacuum” of space. This means that space can be considered as a kind of transparent field with elastic properties. Figure 5 shows the curvature of space.

If space curves, then an inward normal stress “–P” is generated (Figure 5). This normal stress, i.e. surface force serves as a sort of pressure field. It is now understood that the membrane force on the curved surface and each principal curvature generates the normal stress “–P” with its direction normal to the curved surface as a surface force. The normal stress “–P” acts towards the inside of the surface as shown in Figure 5.

\[ -P = N \cdot (2R^{00})^{1/2} = N \cdot (1/R_1 + 1/R_2) \]  

(2)

where N is the line stress, \( R_1, R_2 \) are the radius of principal curvature of curved surface, and \( R^{00} \) is the major component of spatial curvature.

A thin-layer of curved surface will take into consideration within a spherical space having a radius of \( R \) and the principal radii of curvature that are equal to the radius \( (R_1=R_2=R) \). Since the membrane force \( N \) (serving as the line stress) can be assumed to have a constant value, Eq (2) indicates that the curvature \( R^{00} \) generates the inward normal stress “–P” of the curved surface. The inwardly directed normal stress serves as a pressure field [1, 12-17].
2.3 Collimation Mechanism Induced by Spatial Curvature around Magnetic Field

Another collimation mechanism is summarized as follows.

1) On the supposition that space is an infinite continuum, continuum mechanics can be applied to the so-called “vacuum” of space. This means that space can be considered as a kind of transparent elastic field. That is, space as a vacuum performs the motion of deformation such as expansion, contraction, elongation, torsion and bending. We can regard space as an infinite elastic body like rubber.

2) As previously mentioned, from General Relativity, the major component of curvature of space \( R^{00} \) can be produced by not only mass density but also magnetic field \( B \) as follows (See Chapter 1 and Appendix A: Curvature Control by Magnetic Field)

\[
R^{00} = \frac{4\pi G}{\mu_0 c^4} \cdot B^2 = 8.2 \times 10^{-38} \cdot B^2
\]

Eq.(3) indicates that the major component of spatial curvature can be controlled by magnetic field \( B \).

3) If space curves, then an inward normal stress “–P” is generated (See 2.2).

This normal stress, i.e. surface force serves as a sort of a pressure field.

\[
-P = N \cdot (2R^{00})^{1/2} = N \cdot (1/R_1 + 1/R_2)
\]

where \( N \) is the line stress of membrane of curved surface, \( R_1, R_2 \) are the radii of principal curvature of curved surface.

Next, we describe the pressure of the cylindrical space from the shape of the astrophysical jet. Figure 6 shows the spatial pressure induced by magnetic field acting on the surface of astrophysical jet.
Figure 7 shows the spatial pressure as an inward normal stress “−P” about spherical curved space (a) and cylindrical curved space (b).

As shown in Fig. 7 (b), the principal radii of curvature \( R_1 = \infty \) and \( R_2 = a \) yield the following:

\[
- P = N \cdot (2R^00)^{1/2} = N \cdot (1/R_1 + 1/R_2) = N \cdot 1/R_2 = N \cdot 1/a
\]  

(5)

From Eq.(A 15) in Appendix A,

\[
a = \frac{c^2}{\sqrt{G \cdot B}} \left( \frac{4\pi}{\mu_0} \right) \cdot \frac{1}{B}
\]  

(6)

Then we get

\[
- P = N \cdot \frac{1}{a} = \sqrt{\frac{4\pi G}{\mu_0 c^4}} N \cdot B
\]  

(7)

Accordingly, spatial pressure (normal stress) from the concentric spatial surface surrounding the jet collimates the jet.

Since the present space is rigid, the line stress of space “N” seems to be expected as large value.

As mentioned before, magnetic force model (magnetic centrifugal force and magnetic pressure) is regarded as the most dominant theory which solves the two problems of astrophysical jet acceleration and collimation at the same time. That is, the accretion disk generates a helical magnetic field by twisting the magnetic field lines, accelerates by magnetic force, and narrows the jet by magnetic tension (pinch).
On the other hand, since the magnetic field is present in the astrophysical jet, a spatial curvature is generated induced by magnetic field in the surrounding space; a spatial pressure in space equivalent to the gravitational effect is generated in the direction of the interior of the astrophysical jet as well as the pinch force from the outer circumferential surface of the astrophysical jet.

Accordingly, although its effect may be small than magnetic pressure, another collimation mechanism of astrophysical jet can be possible to exist.

### 3. Application for Gravitational Wave Generation

#### 3.1. Gravitational Wave

Gravitational wave is a strain wave of space and this can be also generated by strong magnetic field. The principle of gravitational wave generation using the fluctuation in strain field induced by magnetic field is introduced. Here, strain is the strain which is defined on the continuum mechanics [18, 19].

However, the generation of gravitational wave is very weak, so the occurrence of the ground is impossible from a strength standpoint. Possible gravitational wave source, at present, are limited to the celestial phenomenon with an abrupt change.

That is, although they are the burst gravitational wave to be released during the supernova explosion, the star of gravitational collapse and the continuous gravitational wave released by binary pulsar, their strength are even so very weak. The strength of the gravitational wave that reaches the Earth is said to be about $h = 10^{-29}$ to $h = 10^{-25}$.

Here “$h$” denotes the amplitude of the average gravitational wave in general.

Although various gravitational wave antennas had been developed, the detection sensitivity was about $h = 1.1 \times 10^{-15} - 2.1 \times 10^{-18}$ (in 1989). Gravitational wave detection sensitivity of the current world’s highest sensitivity is now $h = 10^{-22}$.

In the following section, the gravitational wave generation method using magnetic field is introduced.
3.2. Gravitational Wave as Strain Wave of Space

In the first place, we explain about the strain of space [1, 16, 17].

Space is an infinite continuum and its structure is determined by Riemannian geometry. We a priori accept that the nature of actual physical space is a four-dimensional Riemann space, that is, three dimensional space (x=x¹, y=x², z=x³) and one dimensional time (w=ct=x⁰), where c is the velocity of light. These four coordinate axes are denoted as x⁰ (i=0, 1, 2, 3).

The square of the infinitesimal distance “ds” between two infinitely proximate points x¹ and x¹+dx¹ is given by equation of the form:

\[ ds^2 = g_{ij} dx^i dx^j \]  \hspace{1cm} (8)

where \( g_{ij} \) is a metric tensor.

The metric tensor \( g_{ij} \) determines all the geometrical properties of space and it is a function of this space coordinate. In Riemann space, the metric tensor \( g_{ij} \) determines a Riemannian connection coefficient \( \Gamma_{ijk} \), and further more determines the Riemann curvature tensor \( R_{ijk} \) or \( R_{ij} \), thus the geometry of space is determined by metric tensor.

An external physical action such as the existence of mass energy or electromagnetic energy yields the structural deformation of space. In the deformed space region, the infinitesimal distance is given by:

\[ ds'^2 = g'_{ij} dx'^i dx'^j \]  \hspace{1cm} (9)

where \( g'_{ij} \) the metric tensor of deformed space region, and we use the convected coordinates ( \( x'^i = x^i \)).

Since the degree of deformation can be expressed as the change of distance between the two points, referring to continuum mechanics, we get:

\[ ds^2 - ds'^2 = g'_{ij} dx^i dx^j - g_{ij} dx^j dx^j = (g'_{ij} - g_{ij}) dx^i dx^j = 2 e_{ij} dx^i dx^j \]  \hspace{1cm} (10)

where \( g'_{ij}, g_{ij} \) is a metric tensor, \( e_{ij} \) is a strain tensor, and \( ds^2 - ds'^2 \) is the square of the infinitesimal distance between two infinitely proximate points \( x' \) and \( x'+dx' \).

Hence the degree of geometrical and structural deformation can be expressed by the quantity denoted change of metric tensor, i.e., strain tensor.

Eq.(10) indicates that a certain geometrical structural deformation of space is shown by the concept of strain. In essence, the change of metric tensor \( (g'_{ij} - g_{ij}) \) due to the existence of mass energy or electromagnetic energy tensor produces the strain field \( e_{ij}. \)

Namely, a certain structural deformation is described by strain tensor \( e_{ij}. \) From Eq.(10), the strain of space is described as follows:

\[ e_{ij} = 1/2 \cdot (g'_{ij} - g_{ij}) \]  \hspace{1cm} (11)
Now, the metric of flat space where substances are not present is Minkowski metric $\eta_{ij}$ . In the case of small curvature limit, that is, supposing that the deviation from the flat space is small enough, the metric of space $g_{ij}$ is given by
\[ g_{ij} = \eta_{ij} + h_{ij} \] (12)
where $h_{ij}$ is the deviation from the flat space.

From Eq.(12) and (11), $g_{ij} - \eta_{ij} = h_{ij} = (g_{ij} - \eta_{ij}) = 2e_{ij}$, then we obtain
\[ e_{ij} = \frac{1}{2} h_{ij} \] (13)

As is well known in General Relativity, using gravitational field equation, Eq.(14) is obtained from the linear equations to the primary of small amount of $h_{ij}$:
\[ \frac{1}{2} \, \Box \, h_{ij} = \Box \, e_{ij} = -\frac{8\pi G}{c^4} \cdot T_{ij} \] (14)

Namely, the wave equation is obtained by the usual method in General Relativity:
\[ \Box \, e_{ij} = \left( \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) e_{ij} = -\frac{8\pi G}{c^4} \cdot T_{ij} \] (15)

where $\Box$ is the d’Alembert operator (represented by a box: $\Box$), $G$ is the gravitational constant, $c$ is the light speed and $T_{ij}$ is the energy momentum tensor.

Eq.(15) has a wave solution to propagate at the speed of light, strain of the space $e_{ij}$ in which this propagation is a gravitational wave.

In flat space with no matter, since the energy momentum tensor $T_{ij}$ is zero, the wave equation becomes
\[ \frac{1}{2} \, \Box \, h_{ij} = \Box \, e_{ij} = 0 \] (16)

Also, the metric tensor in flat space is Minkowski metric at any location, i.e. $\eta_{ij} = \eta_{ij}$, so $e_{ij} = 0$ yields the divergence of displacement vector “$u$”, i.e., volume strain is zero: $e_{11}+e_{22}+e_{33} = 0$.

There exists the following wave equation of space-time satisfying $\text{div} \, u = 0$, that is, $\nabla^2 u - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{u} = 0$. And also, this wave motion requires the transversal wave. The strain wave of space is essentially a transverse wave.
3.3. Generation of Gravitational Wave by Magnetic Field

Since the major component of spatial curvature $R^{00}$ is independent of the cause of material energy and electromagnetic energy as a source, the following equations are obtained:

$$R^{00} = \frac{4\pi G}{\mu_0 c^4} \cdot B^2 = \frac{8\pi G}{c^2} \cdot \rho . \quad (17)$$

This yields

$$\frac{B^2}{2\mu_0} = \rho c^2 = w \quad (18)$$

From Eq.(14) and electromagnetic energy tensor $M_{ij}$ and Ricci tensor $R_{ij}$ (see APPENDIX A Eq.(A8)), we have:

$$\frac{1}{2} \square h_{ij} = -\frac{8\pi G}{c^4} \cdot T_{ij} \quad (19)$$

$$R_{ij} = -\frac{8\pi G}{c^4} \cdot M_{ij} \quad (20)$$

then we get

$$\frac{1}{2} \cdot \square h_{ij} = R_{ij} \quad (\square e_{ij} = R_{ij} ) \quad (21)$$

In the wave equation Eq.(21), the time variation of $h_{ij}$ depends on the temporal fluctuation of curvature $R_{ij}$ of space. That is, since the curvature of space is varied in terms of time, the strain field of space $e_{ij} = \frac{1}{2} h_{ij}$ is temporally change, then the strain wave of space due to the fluctuation in strain field is generated.

Concerning the solution of the wave equation Eq.(21) for any of $R_{ij}$, the wave solution of outward is determined by the delay potential as with the electromagnetic wave:

$$h_{ij} = \frac{4G}{c^3 r} \int R_{ij}(ct-r,x)dV \quad (22)$$

As is well known in gravitational wave, in the wave equation in the empty space-time with no matter, the deviation from the flat space $h_{ij}$ has 10 independent components.

Then let’s think about gravitational wave radiation. In the case of $i=j=0$, Eq.(21) becomes

$$\frac{1}{2} \cdot \square h_{00} = R_{00} \quad (23)$$
On the other hand, we have

\[
\frac{1}{2} \left( c^2 h_{00} - \frac{1}{c^2} \frac{\partial^2 h_{00}}{\partial t^2} \right) = R_{00}
\] (24)

We have the relation between magnetic field \( B \) and mass density \( \rho \) from Eq.(18):

\[
B = \sqrt{2} \mu_0 \cdot c \sqrt{\rho}
\] (29)

So, although it may be indirect method, we can predict the intensity of required magnetic field \( B \) to detect gravitational wave using the gravitational wave antenna. Since near-field close to a gravitational wave sources do not have a property as a wave, in order to capture as radiation, it is necessary to measure at least one wave length of about away from the gravitational wave source.

Then, applying ON/OFF pulses of magnetic field \( B \), we can get time change of major component of spatial curvature \( R_{00} \):

\[
R_{00} = \frac{4\pi G}{\mu_0 c^4} \cdot B^2 = 8.2 \times 10^{-38} \cdot B^2 \quad (B \text{ in Tesla})
\] (25)

Namely, the fluctuation in strain field of space is conclusively obtained by fluctuation of the magnetic field. Further, it is possible to generate a gravitational wave directly by varying equivalent magnetic field just as the distribution control of quadrupole moment tensor of the object.

Even if the gravitational wave does not generate by magnetic field as wave, fluctuation in gravitational field can be generated by magnetic field in the vicinity.

Energy flux of gravitational wave \( S \) (the flow of energy per unit time across a unit area perpendicular to the flow) \( S \) is given by

\[
S = \frac{c^5 k^2}{32\pi G} \cdot h^2 = \frac{3\pi f^2}{8G} \cdot h^2
\] (26)

where \( k = \omega / c = 2\pi f / c \), \( h \) is the amplitude of the average gravitational wave.

Meanwhile, the energy flux of gravitational wave \( S \) is obtained by the following equation to be received from the gravitational wave source (gravitational wave emission power \( P \) (W)) at a distance “\( r \)” away from Earth (with Eq.(26)):

\[
S = \frac{P}{4\pi r^2} = \frac{3\pi f^2}{8G} \cdot h^2
\] (27)

Then we get the \( h \) as the amplitude of the average gravitational wave as follows:

\[
h = \sqrt{\frac{2G / c^3}{\pi f}} \cdot \sqrt{\frac{P}{r}}
\] (28)

We have the relation between magnetic field \( B \) and mass density \( \rho \) from Eq.(18):
Since the curvature of space can be also generated by magnetic field, pseudo-gravitational wave as the fluctuation in strain field (change the gravitational field) or gravitational wave as strain wave of space can be generated. The amplitude $h$ of gravitational wave can be controlled by magnetic field, and also a fairly high frequency of gravitational wave can be possible.

The exact order is currently under consideration, but the amplitude of the gravitational wave $h \approx 10^{-24}$ is expected to be generated by the magnetic field of 20,000 Tesla.

3.4. Trends of Latest Gravitational Wave Detection

On February 11, 2016, it was published the analysis of advanced LIGO event GW150914 and it was claimed that it was the first direct observation for gravitational waves and black holes at the same time. The signal was named GW150914. Also, on October 16, 2017, the second detection of gravitational wave has confirmed. This is named GW170817. It was the first observation as a gravitational wave by coalescence of neutron star.

In general, gravitational wave amplitude to be expected is said to be very small, i.e., $h \approx 10^{-29} - 10^{-25}$.

On the other hand, during LIGO’s fifth Science Run in November 2005, sensitivity reached the primary design specification of a detectable strain of one part in $h = 10^{-21}$ over a 100 Hz bandwidth. LIGO has already attained the detection sensitivity of $h = 10^{-22}$. In LIGO 100Hz-10 kHz, currently, the world’s highest sensitivity. Also, the detection sensitivity of Advanced LIGO is said to be $h = 10^{-23}$.

Also in Japan, there exist TAMA 300 and KAGRA. TAMA 300 was a gravitational wave detector located at the Mitaka campus of the National Astronomical Observatory of Japan, and is currently working on. The Kamioka Gravitational Wave Detector (KAGRA) is a project of the gravitational wave and is currently developed. The target detection sensitivity of KAGRA is $h \approx 10^{-23} - 10^{-24}$.

Since the current sensitivity of gravitational wave antenna has already reached $h = 10^{-22}$, the amplitude of the gravitational wave $h = 10^{-22}$ is expected to be generated by the magnetic field of 200,000 Tesla, further $h \approx 10^{-24}$ is expected to be generated by the magnetic field of 20,000 Tesla (exact order is currently under consideration). Namely, if KAGRA will attain $h \approx 10^{-24}$ or less of the detection sensitivity in the near future, the gravitational wave or fluctuation in gravitational field generated by the magnetic field of 20,000 Tesla may be directly detected.

4. Application for Space Propulsion System

4.1 Space Propulsion Principle

When the curved surfaces are included in a great number, some type of unidirectional pressure field is formed. Figure 8 shows Curvature of Space. A region of curved space is made of a large number of curved surfaces and they form the field as a unidirectional surface force (i.e. normal stress). Since the field of the surface force is the field of a kind of force, the force accelerates matter in the field, i.e. we can regard the field of the surface force as the acceleration field. A large number of curved thin layers form the unidirectional acceleration field (Figure 8(b)). Accordingly, the spatial curvature $\kappa^{00}$ produces the acceleration field $\alpha$. Therefore, the curvature of space plays a significant role to generate pressure field (see Eq.(2)).

Figure 9 shows the basic propulsion principle of common to all kinds of field propulsion system. The term of field propulsion means to propel by utilizing the space field around the spaceship. The field propulsion principle is based on the assumption that space as a vacuum possesses a substantial physical structure. Field propulsion is propelled receiving the propulsive force (i.e., thrust) arises from the interaction of the substantial physical structure. Typical field propulsion is space drive propulsion system.

As shown in Figure 9, the propulsion principle of field propulsion system is not momentum thrust but pressure thrust induced by a pressure gradient (or potential gradient) of the space-time field (or vacuum field) between bow and
stern of a spaceship. Since the pressure of the vacuum field is high in the rear vicinity of spaceship, the spaceship is pushed from the vacuum field. Pressure of vacuum field in the front vicinity of spaceship is low, so the spaceship is pulled from the vacuum field. In the front vicinity of spaceship, the pressure of vacuum field is not necessarily low but the ordinary vacuum field, that is, just as only a high pressure of vacuum field in the rear vicinity of spaceship. The spaceship is propelled by this distribution of pressure of the vacuum field. Vice versa, it is the same principle that the pressure of vacuum field in the front vicinity of spaceship is just only low and the pressure of vacuum field in the rear vicinity of spaceship is ordinary. In any case, the pressure gradient from the vacuum field (potential gradient) is formed over the entire range of the spaceship, so that the spaceship is propelled by pushing from the pressure gradient resulting from the vacuum field.

4.2. Acceleration induced by Spatial Curvature

A massive body causes the curvature of space-time around it, and a free particle responds by moving along a geodesic line in that space-time. The path of free particle is a geodesic line in space-time and is given by the following geodesic equation;

\[ \frac{d^2 x^i}{d\tau^2} + \Gamma^i_{jk} \frac{dx^j}{d\tau} \frac{dx^k}{d\tau} = 0 \]  \hspace{1cm} (30)
where $\Gamma_{jk}^i$ is Riemannian connection coefficient, $\tau$ is proper time, $x^i$ is four-dimensional Riemann space, that is, three dimensional space ($x=x^1, y=x^2, z=x^3$) and one dimensional time ($w=ct=x^0$), where $c$ is the velocity of light. These four coordinate axes are denoted as $x^i$ ($i=0, 1, 2, 3$).

Proper time is the time to be measured in a clock resting for a coordinate system. We have the following relation derived from an invariant line element $ds^2$ between Special Relativity (flat space) and General Relativity (curved space):

$$d\tau = \sqrt{-g} \; dx^0 = \sqrt{-g} \; cdt$$

(31)

From Eq.(30), the acceleration of free particle is obtained by

$$\alpha^i = \frac{d^2 x^i}{d\tau^2} = -\Gamma^i_{jk} \frac{dx^j}{d\tau} \frac{dx^k}{d\tau}$$

(32)

As is well known in General Relativity, in the curved space region, the massive body "$m$ (kg)" existing in the acceleration field is subjected to the following force $F^i$ (N) :

$$F^i = m\Gamma^i_{jk} \frac{dx^j}{d\tau} \frac{dx^k}{d\tau} = m\sqrt{-g} c^2 \Gamma_{jk}^i u^j u^k = m\alpha^i,$$

(33)

where $u^j, u^k$ are the four velocity, $\Gamma_{jk}^i$ is the Riemannian connection coefficient, and $\tau$ is the proper time.

From Eqs.(32),(33), we obtain:

$$\alpha^i = \frac{d^2 x^i}{d\tau^2} = -\Gamma^i_{jk} \frac{dx^j}{d\tau} \frac{dx^k}{d\tau} = -\sqrt{-g} \; c^2 \Gamma_{jk}^i u^j u^k$$

(34)

Eq.(34) yields a more simple equation from the condition of linear approximation, that is, weak-field, quasi-static, and slow motion (speed $v << \text{speed of light} \; c \cdot U^0 \approx 1$):

$$\alpha^i = -\sqrt{-g} \; c^2 \Gamma_{00}^i$$

(35)

On the other hand, the major component of spatial curvature $K_{00}$ in the weak field is given by

$$K_{00} \approx R_{00} = R_{\mu\nu\rho\sigma}^\mu = \partial_0 \Gamma_{\mu\nu}^\mu - \partial_\mu \Gamma_{0\nu}^\mu + \Gamma_{0\mu}^\nu \Gamma_{\nu0}^\mu - \Gamma_{\nu0}^\nu \Gamma_{0\mu}^\mu$$

(36)

In the nearly Cartesian coordinate system, the value of $\Gamma_{ij\rho}$ are small, so we can neglect the last two terms in Eq.(36), and using the quasi-static condition we get
From Eq.(37), we get formally

\[ \Gamma^i_{00} = -\int R^0_0(x') dx' \]  

Substituting Eq.(38) into Eq.(35), we obtain

\[ \alpha^i = \sqrt{-g_{00}} c^2 \int R^0_0(x') dx' \]  

Accordingly, from the following linear approximation scheme for the gravitational field equation: (1) weak gravitational field, i.e. small curvature limit, (2) quasi-static, (3) slow-motion approximation (i.e., \( v/c < 1 \)), and considering range of curved region, we get the following relation between acceleration of curved space and curvature of space:

\[ \alpha^i = \sqrt{-g_{00}} c^2 \int_a^b R^0_0(x') dx' , \]  

where \( \alpha^i \) : acceleration (m/s²), \( g_{00} \) : time component of metric tensor, a-b: range of curved space region (m), \( x^i \): components of coordinate (\( i=0,1,2,3 \)), \( c \): velocity of light, \( k^{00} \) : major component of spatial curvature(1/m²).

Eq.(40) indicates that the acceleration field \( \alpha^i \) is produced in curved space. The intensity of acceleration produced in curved space is proportional to the product of spatial curvature \( k^{00} \) and the length of curved region. Eq.(33) yields more simple equation from above-stated linear approximation (\( u^0 \approx 1 \)),

\[ F^i = m\sqrt{-g_{00}} c^2 \Gamma^i_{00} u^0 = m\sqrt{-g_{00}} c^2 \Gamma^i_{00} = ma^i = m\sqrt{-g_{00}} c^2 \int_a^b R^0_0(x') dx' \]  

Setting \( i=3 \) (i.e., direction of radius of curvature: \( r \)), we get Newton’s second law:

\[ F^3 = F = ma = m\sqrt{-g_{00}} c^2 \int_a^b R^0_0(r) dr = m\sqrt{-g_{00}} c^2 \Gamma^3_{00} . \]  

The acceleration \( \alpha \) of curved space and its Riemannian connection coefficient (\( \Gamma^3_{00} \)) are given by:

\[ \alpha = \sqrt{-g_{00}} c^2 \Gamma^3_{00} , \quad \Gamma^3_{00} = -\frac{g_{00,3}}{2g^{33}} \]  

where \( c \): velocity of light, \( g_{00} \) and \( g_{33} \): component of metric tensor, \( g_{00,3} \): \( \partial g_{00}/\partial x^3 = \partial g_{00}/\partial r \). We choose the spherical coordinates “\( ct=x^0, r=x^1, \theta=x^2, \phi=x^3 \)” in space-time. The acceleration \( \alpha \) is represented by the equation both in the differential form and in the integral form. Practically, since the metric is usually given by the solution of gravitational field equation, the differential form has been found to be advantageous.
The principle of this space propulsion system is derived from General Relativity and the theory of continuum mechanics. We assume that the so-called “vacuum” of space acts as an infinite elastic body like rubber. The curvature of space plays a significant role for propulsion theory. The acceleration performance of this system is found by the solution of the gravitational field equation, such as the Schwarzschild solution, Reissner- Nordstrom solution, Kerr solution, and de Sitter solution [1, 20-24].

Figure 10: (a) A condensed summary of space drive propulsion principle; (b) motion of the spaceship using computer graphics: spaceship is pushed from the expanded space and advances forward.

SPACE DRIVE PROPULSION SYSTEM

- Curvature of SPACE \( (R^0) \) plays a significant role for propulsion theory (Y. Minami: 1988).

\[
F^i = m \sqrt{-g_{00}} c^2 \Gamma^i_{00} = m \alpha^i = m \sqrt{-g_{00}} c^2 \int_a^b R^{00} (x') dx'
\]

\[
R^{00} = \frac{4 \pi G}{\mu_0} \frac{B^2}{c^4} \quad \text{Both strength of curvature and its extent (volume) are important.}
\]

- Acceleration induced by de Sitter solution is found in 1996 by Minami: constant acceleration \( a \) (i.e. no tidal force inside of the starship).

\[
a = \frac{2 \pi G \lambda}{3c^2} \Phi_0^4 = 1.6 \times 10^{-27} \Phi_0^4
\]

\( \Phi_0 \): non-zero vacuum expectation value of field

In conclusion, a condensed summary of the propulsion principle of space drive propulsion system is shown as Figure 10. The application of mechanism of accretion disk and astrophysical jet around black holes will lead to the
concrete system design of propulsion engine and power source installed in space drive propulsion system [8, 23].

4.3 Strong Magnetic Field Generation by Magnetic Field Line Break-Reconnection

In astrophysics, magnetic field reconnection can be working in all areas of space at all times. It works not only on the surface of the sun and in the sun’s solar flares but also in rotating fields of accretion disks in space. As is well known, magnetic field reconnection will provide copious productions of electron-positron charged particles, and will produce so much energy. The magnetic reconnection is considered to be promising as a solar flare energy release mechanism, but it seems not necessarily clear.

When the magnetic flux lines are snapped, the virtual particle of the vacuum will become unstable at that gap and will break down to transform those virtual particles into real electrically charged particles. When the breakdown in the vacuum occurs, avalanches of electron-positron pairs will be produced out of that of space. In space the potential drop threshold for when this effect takes place around the breaking of magnetic flux lines is said to be about $10^{12}$ volts. This particle producing mechanism can be found around Sun, where the chromosphere continually produces electrons and positrons by this method and then has to eject out from it around a million tons of those charged particles every second.

This magnetic field line reconnection process is working throughout the whole cosmos, and in active galactic nuclei which are some of all the astrophysical jets. The magnetic field reconnection system is regarded as one of the most efficient production method of charged particles in the galaxy. Around accretion disk the shearing-reconnection of strong magnetic field produces a dynamo effect which gives a rapid amplification of any incoming and smaller electrical field of charged particles (seed field). So, they will develop into much larger field and go on to accelerate particles which will collide with other particles, to produce more particles, and more collisions, which subsequently will lead to avalanche productions of more electron-positron pairs.

Anyway, electron-positron pair production comes from the breaking and reconnection of magnetic field lines. The breaking and remaking of magnetic field lines produces and then amplifies amounts of electrons and positrons from what some have called the ‘empty’ vacuum of space. After all, key is energy generation by magnetic field breaking and magnetic reconnection. Because mass production of charged particles by electron avalanche phenomenon and generation of electron positron pairs accompanying this can be utilized. The generation of a large amount of charged particles bring about the generation of a large current, and it is possible to generate a strong magnetic field from this large current. A strong magnetic field is indispensable for energy generation and spatial curvature generation.

5. Conclusion

The role and influence of gravitation on phenomena of astrophysics is immeasurable. It is well known that gravitation is generated by curvature of space. The curvature of the space is generated not only by object mass but also by magnetic field. We introduced three applications using spatial curvature induced by magnetic field, that is, another collimation mechanism of astrophysical jet, gravitational wave generation, advanced space propulsion by space drive. Three applications based on spatial curvature generation by magnetic fields are probably new attempts.

APPENDIX A: Curvature Control by Magnetic Field

Let us consider the electromagnetic energy tensor $M^{ij}$. In this case, the solution of metric tensor $g_{ij}$ is found by

$$R^{ij} - \frac{1}{2} g^{ij} R = -\frac{8\pi G}{c^4} M^{ij}, \quad (A.1)$$

Eq.(A.1) determines the structure of space due to the electromagnetic energy.
Here, if we multiply both sides of Eq.(A.1) by $g_{ij}$, we obtain

$$g_{ij} \left( R^{ij} - \frac{1}{2} g^{ij} R \right) = g_{ij} R^{ij} - \frac{1}{2} g_{ij} g^{ij} R = R - \frac{1}{2} A R = -R \quad , \quad (A.2)$$

$$g_{ij} \left( -\frac{8\pi G}{c^4} M^{ij} \right) = -\frac{8\pi G}{c^4} g_{ij} M^{ij} = -\frac{8\pi G}{c^4} M_i = -\frac{8\pi G}{c^4} M \quad , \quad (A.3)$$

The following equation is derived from Eqs.(A.2) and (A.3)

$$R = \frac{8\pi G}{c^4} M \quad . \quad (A.4)$$

Substituting Eq.(A.4) into Eq.(A.1), we obtain

$$R^{ij} = -\frac{8\pi G}{c^4} M^{ij} + \frac{1}{2} g^{ij} R = -\frac{8\pi G}{c^4} \left( M^{ij} - \frac{1}{2} g^{ij} M \right) \quad . \quad (A.5)$$

Using antisymmetric tensor $f_{ij}$ which denotes the magnitude of electromagnetic field, the electromagnetic energy tensor $M^{ij}$ is represented as follows;

$$M^{ij} = -\frac{1}{\mu_0} \left( f^{ij} f_{ij} - \frac{1}{4} g^{ij} f_{ij} \right) , f^{ij} = g^{ia} g^{jb} f_{aj} \quad . \quad (A.6)$$

Therefore, for M, we have

$$M = M_i = g_{ij} M^{ij} = -\frac{1}{\mu_0} \left( g_{ij} f^{i\rho} f_{j\rho} - \frac{1}{4} g_{ij} g^{ij} f_{\alpha\beta} f_{\alpha\beta} \right)$$

$$= -\frac{1}{\mu_0} \left( f^{i\rho} f_{i\rho} - \frac{1}{4} f^{\alpha\beta} f_{\alpha\beta} \right) = -\frac{1}{\mu_0} \left( f^{i\rho} f_{i\rho} - f^{i\rho} f_{i\rho} \right) = 0 \quad (A.7)$$

Accordingly, substituting $M = 0$ into Eq.(A.5), we get

$$\mathcal{R}^{ij} = -\frac{8\pi G}{c^4} M^{ij} \quad (A.8)$$

Although Ricci tensor $\mathcal{R}^{ij}$ has 10 independent components, the major component is the case of $i = j = 0$, i.e., $\mathcal{R}^{00}$. Therefore, Eq.(A.8) becomes.

$$\mathcal{R}^{00} = -\frac{8\pi G}{c^4} M^{00} \quad (A.9)$$
On the other hand, 6 components of antisymmetric tensor $\tilde{f}_{ij} = -f_{ji}$ are given by electric field $E$ and magnetic field $B$ from the relation to Maxwell’s field equations

\[
\begin{align*}
    f_{10} &= -f_{01} = \frac{1}{c} E_x, \\
    f_{20} &= -f_{02} = \frac{1}{c} E_y, \\
    f_{30} &= -f_{03} = \frac{1}{c} E_z, \\
    f_{12} &= -f_{21} = B_z, \\
    f_{23} &= -f_{32} = B_x, \\
    f_{31} &= -f_{13} = B_y.
\end{align*}
\quad (A.10)
\]

Substituting Eq.(A.10) into Eq.(A.6), we have

\[
M^{00} = -\left(\frac{1}{2} \varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2 \right) z - \frac{1}{2\mu_0} B^2
\quad (A.11)
\]

Finally, from Eqs.(A.9) and (A.11), we get

\[
R^{00} = \frac{4\pi G}{\mu_0 c^4} B^2 = 8.2 \times 10^{-38} \text{, } B^{\text{in Tesla}} \quad (A.12)
\]

where we let $\mu_0 = 4\pi \times 10^{-7} \text{ (H} / \text{m})$, $\varepsilon_0 = 1/(36\pi) \times 10^{-9} \text{ (F} / \text{m})$, $c = 3 \times 10^8 \text{ (m} / \text{s})$,

$G = 6.672 \times 10^{-11} \text{ (N} \cdot \text{m}^2 / \text{kg}^2)$, $B$ is a magnetic field in Tesla and $R^{00}$ is a major component of spatial curvature \(1/\text{m}^2\).

The relationship between curvature and magnetic field was derived by Minami and introduced it in 16th International Symposium on Space Technology and Science (1988) [1].

Eq.(A12) is derived from general method.

On the other hand, Levi-Civita also investigated the gravitational field produced by a homogeneous electric or magnetic field, which was expressed by Pauli [2]. If $x^3$ is taken in the direction of a magnetic field of intensity $F$ (Gauss unit), the square of the line element is of the form;

\[
d s^2 = \left(dx^1\right)^2 + \left(dx^2\right)^2 + \left(dx^3\right)^2 + \frac{\left(x^1 dx^1 + x^2 dx^2\right)^2}{a^2 - r^2} + \left[c_1 \exp\left(x^3 / a\right) + c_2 \exp\left(-x^3 / a\right)\right] \left(dx^4\right)^2
\quad (A.13)
\]
where \( r = \sqrt{\left(x^1\right)^2 + \left(x^2\right)^2} \), \( c_1 \) and \( c_2 \) are constants, \( a = \frac{c^2}{\sqrt{kF}} \), \( k \) is Newtonian gravitational constant\((G)\), and \( x'...x' \) are Cartesian coordinates \((x'...x'=\text{space}, x'=ct)\) with orthographic projection.

The space is cylindrically symmetric about the direction of the field, and on each plane perpendicular to the field direction the same geometry holds as in Euclidean space on a sphere of radius \( a \), that is, the radius of curvature \( a \) is given by

\[
a = \frac{c^2}{\sqrt{kF}} \quad (A.14)
\]

Since the relation of between magnetic field \( B \) in SI units and magnetic field \( F \) in CGS Gauss units are described as follows: \( B \frac{4\pi}{\mu_0} \Leftrightarrow F \), then the radius of curvature “\( a \)” in Eq.(A14) is expressed in SI units as the following (changing symbol, \( k \rightarrow G, F \rightarrow B \)):

\[
a = \frac{c^2}{\sqrt{G F}} = \frac{c^2}{\sqrt{G.B \frac{4\pi}{\mu_0}}} \approx \left( 3.484 \times 10^{18} \frac{1}{B \text{ meters}} \right) \quad (A15)
\]

While, scalar curvature is represented by

\[
R^{00} \approx R = \frac{1}{a^2} = \frac{GB^2 \frac{4\pi}{\mu_0}}{c^4} = \frac{4\pi G}{\mu_0 c^4} B^2
\]

which coincides with (A.12).

References


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